A low diffusive Lagrange+Remap scheme for the simulation of violent air-water free-surface flows. Applications to dam-break and sloshing events.

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Introduction

- Simulation of free surface flows
  - Dam-break events, Offshore pipes, Sloshing events, Effects of tidal bore…
- All these examples imply a large deformation of the interface between the two fluids: **an accurate method to follow the interface is required.**

We focus on:

- Air/water simulations: two fluids with a high ratio of density
- Physical effects that have to be taken into account:
  - Effects of the compressibility of the air
  - Since $\rho_l / \rho_g \simeq 1000$, Archimède's buoyancy effects
  - Air pocket formation

- We neglect:
  - Surface tension
  - Viscosity of the fluids

Choice of the modelisation

- Choice of the **numerical method**
  - Particle methods (mesh-less)
    - Simulation using SPH (Smoothed Particle Hydrodynamics) techniques or MPS (Moving particle semi-implicit)
    - ☺ Without numerical diffusion at the interface
    - ☹ Need an high number of particles → high computational cost
  - Numerical methods with a mesh
    - With interface capturing or interface tracking
    - ☺ Able to follow large deformation of the interface
    - ☹ Diffusion at the interface.
- Treatment of the liquid which is quasi-incompressible
  - as a compressible fluid with a stiff equation of state
    - ☺ Less computational cost (than solving the Poisson equation)
    - ☹ High celerity of sound which leads to a reduction of the time step

Choice of the EOS for each fluid

- For the air, we use an isentropic **perfect gas law**:

$$p_g(\rho_g) = P_0 \left(\frac{\rho_g}{\rho_{g0}} \right)^{\gamma_g} \qquad \gamma_g = 1.4,\ \rho_{g_0} = 1.28,\ P_0 = 10^5$$

- For the water:
  - We use an isentropic **modified Taït EOS** for the liquid

$$p_l(\rho_l) = P_0 \left(1 + K \left(\left(\frac{\rho_l}{\rho_l^0} \right)^{\gamma_l} - 1 \right) \right) \qquad K = \frac{\rho_l^0 c_s^{\,2}}{P_0 \gamma_l} \quad \text{Bulk modulus}$$

with for instance: $\gamma_l = 7,\ \rho_l^0 = 1000,\ \mathrm{c}_s = 350\ \mathrm{m.s}^{-1},\ (K = 175)$

Weakly compressible: if $P = \dfrac{P_0}{2},\ \rho = 999.59$

System to solve

A four equation volume-averaged air-water model with pressure equilibrium closure

$$\partial_t(\rho) + \nabla \bullet (\rho u) = 0$$

$$\partial_t(\rho c_g) + \nabla(\rho c_g u) = 0$$

$$\partial_t(\rho u) + \nabla \bullet (\rho u \otimes u) + \nabla p = \rho g$$

where

ρ : mean density

u : mean velocity

c_g : concentration of gas

g : gravity

with $\rho = \alpha \rho_g + (1-\alpha)\rho_l$

- α: volume fraction of gas
- ρ_g: density of gas
- ρ_l: density of liquid

and $c_g = \dfrac{\alpha \rho_g}{\rho} = \dfrac{m_g}{m} \in [0,1]$

$m_g = \alpha \rho_g V,$

$m_l = (1-\alpha)\rho_l V$

+

pressure equilibrium assumption to get α and then ρ_g and ρ_l :

$$p = P_g(\rho_g) = P_l(\rho_l)$$

solved → with a Newton algorithm

Numerical scheme

- Solved with a Lagrange-remap scheme: BBC

[WC84] P. Woodward and P. Colella *J. Comp.Phys.*, ***54, 115 (1984).***

Lagrangian phase

$$\rho D_t \frac{1}{\rho} = \nabla \bullet u$$

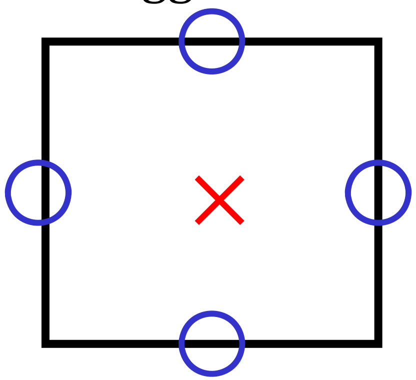
$$\rho D_t u = -\nabla p$$

$$D_t c_g = 0$$

+

Projection on the Eulerian cartesian grid with a **low-difusive procedure** to keep a thin interface

- Staggered mesh



Velocities defined on the edges (○) and other quantities on the center (×).

The lagrangian phase:

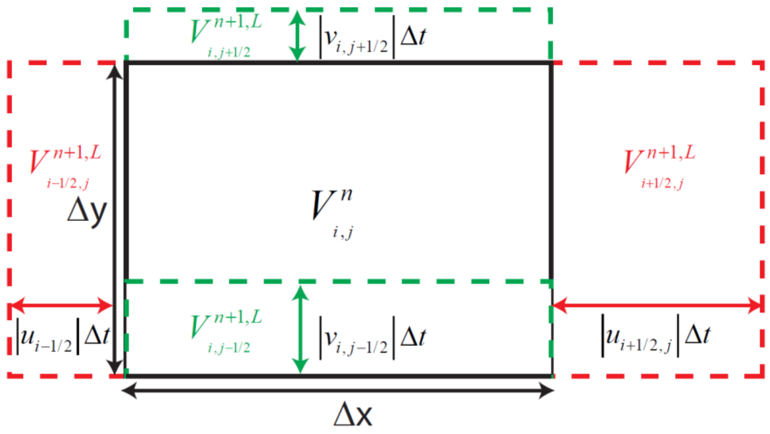
- Use multistep in time (predictor-corrector scheme)
- Centered in space
- Use of a pseudo-viscosity to stabilize the scheme

LOW-DIFFUSIVE PROJECTION

Projection of the quantities

- At the end of the lagrangian phase: **deformed mesh**

New volume of a cell: $V_{i,j}^{n+1,L} = V_{i,j}^{n} + \Delta t \Delta y \left(u_{i+1/2,j}^{n+1/2,L} - u_{i-1/2,j}^{n+1/2,L} \right) + \Delta t \Delta x \left(v_{i,j+1/2}^{n+1/2,L} - v_{i,j-1/2}^{n+1/2,L} \right)$



➡ Projection of the quantities on the cartesian cell of volume ΔxΔy.

- First step: projection along the x direction: on the intermediate volume $V_{i,j}^{n+1,*}$: $V_{i,j}^{n+1,*} = V_{i,j}^{n+1,L} - \Delta t \Delta y \left(u_{i+1/2,j}^{n+1/2,L} - u_{i-1/2,j}^{n+1/2,L} \right)$
- Second step: projection along the y direction:

$$V_{i,j}^{n+1} = V_{i,j}^{n+1,*} - \Delta t \Delta x \left(v_{i,j+1/2}^{n+1/2,L} - v_{i,j-1/2}^{n+1/2,L} \right) = \Delta x \Delta y$$

Projection of the masses at first order

- We focus on the first step of the projection (the second is similar):

Mass of gas in the intermediate volume $V_{i,j}^{n+1,*}$:

$$m_{g\,i,j}^{n+1,*} = m_{g\,i,j}^{n} - \Delta t \Delta y \left[u_{i+1/2,j}^{n+1/2,L} \alpha_{i+1/2,j}^{n+1,L} \rho_{g\,i+1/2,j}^{n+1,L} - u_{i-1/2,j}^{n+1/2,L} \alpha_{i-1/2,j}^{n+1,L} \rho_{g\,i-1/2,j}^{n+1,L} \right] \quad (1)$$

Mass of liquid in the intermediate volume $V_{i,j}^{n+1,*}$:

$$m_{l\,i,j}^{n+1,*} = m_{l\,i,j}^{n} - \Delta t \Delta y \left[u_{i+1/2,j}^{n+1/2,L} (1-\alpha_{i+1/2,j}^{n+1,L}) \rho_{l\,i+1/2,j}^{n+1,L} - u_{i-1/2,j}^{n+1/2,L} (1-\alpha_{i-1/2,j}^{n+1,L}) \rho_{l\,i-1/2,j}^{n+1,L} \right] \quad (2)$$

written in conservative form

where: $V_{i,j}^{n+1,*} = V_{i,j}^{n} + \Delta t \Delta x \left(v_{i,j+1/2}^{n+1/2,L} - v_{i,j-1/2}^{n+1/2,L} \right)$

- How to calculate the values at the edges $\alpha_{i\pm1/2,j}^{n+1,L}$ and $\left(\rho_{g/l}\right)_{i\pm1/2,j}^{n+1,L}$?

First choice: upwind of the quantities.

$$\alpha_{i\pm1/2,j}^{n+1,L,\text{upw}} \rho_{g\,i+1/2,j}^{n+1,L,\text{upw}} = \begin{cases} \alpha_{i,j}^{n+1,L} \rho_{g\,i,j}^{n+1,L} & \text{if } u_{i+1/2,j}^{n+1/2,L} \geq 0 \\ \alpha_{i+1,j}^{n+1,L} \rho_{g\,i+1,j}^{n+1,L} & \text{if } u_{i+1/2,j}^{n+1/2,L} < 0 \end{cases}$$

⇒ stable but **diffusive**.

Low diffusive projection

- **Other choice** for the fluxes: we applied a **low-diffusive procedure** initially developped for the advection of a quantity and extended for a two-fluid sytems

F. Lagoutière, PhD Thesis (2000)

S. Kokh, F. Lagoutière J.C.P 2010

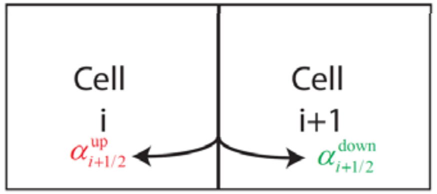
At the interface:
- Continue to upwind ρ_g and ρ_l
- Make a «best» choice for α low diffusive

$$\alpha^{n+1,L}_{i\pm1/2,j}\rho^{n+1,L}_{g\,i+1/2,j} = \alpha^{\mathrm{LowDiff}}_{i+1/2,j}\,\rho^{n+1,L,upw}_{g\,i+1/2,j}$$

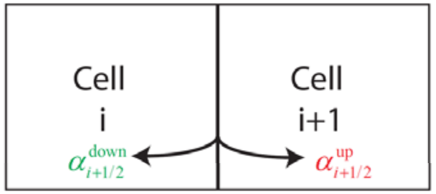
$$\left(1-\alpha^{n+1,L}_{i\pm1/2,j}\right)\rho^{n+1,L}_{l\,i+1/2,j} = \left(1-\alpha^{\mathrm{LowDiff}}_{i+1/2,j}\right)\rho^{n+1,L,upw}_{l\,i+1/2,j}$$

to limit the numerical diffusion

if $u^{n+1,L}_{i+1/2} > 0$



if $u^{n+1,L}_{i+1/2} < 0$



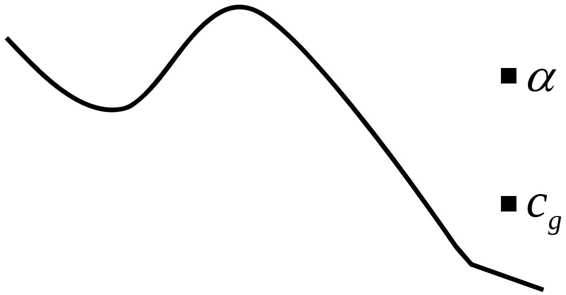
$\alpha^{\mathrm{up}}_{i+1/2}$	stable	diffusive
$\alpha^{\mathrm{down}}_{i+1/2}$	not diffusive	unstable

- **Idea**: take the advantage of both upwind and downwind scheme:

choose $\alpha^{\mathrm{LowDiff}}_{i+1/2} \in \left[\min(\alpha^{\mathrm{down}}_{i+1/2},\alpha^{up}_{i+1/2}),\max(\alpha^{\mathrm{down}}_{i+1/2},\alpha^{up}_{i+1/2})\right]$ to be less diffusive as possible while being stable.

Quantities defining the interface

- The interface is followed thanks to 2 quantities:



  - $\alpha = \dfrac{\text{Vol of gas}}{\text{Vol total}} \in [0,1]$ volume fraction of gaz
  - $c_g = \dfrac{m_g}{m_{\text{tot}}} \in [0,1]$ mass fraction of gaz

$$\alpha\left(\rho_g, \rho_l, c_g\right) = \frac{c_g \rho_l}{c_g \rho_l + \left(1 - c_g\right)\rho_g} \qquad c_g\left(\rho_g, \rho_l, \alpha\right) = \frac{\alpha \rho_g}{\alpha \rho_g + \left(1 - \alpha\right)\rho_l}$$

- The quantity c_g is advected $\partial_t c_g + u \cdot \nabla c_g = 0$

⇨ respect a maximum principle

  - So the projected value $c_{g_{i,j}}^{n+1,*}$ has to respect a discrete maximum principle

- The projected value $\alpha_{i,j}^{n+1,*} \in [0,1]$ thanks to the pressure equilibrium assumption using the projected partial masses $m_{g_{i,j}}^{n+1,*}$ and $m_{l_{i,j}}^{n+1,*}$.

(If $m_{g_{i,j}}^{n+1,*}$ and $m_{l_{i,j}}^{n+1,*} \geq 0.$)

How to choose the flux $\alpha_{i+1/2}$

1) Construct an interval I (**trust interval**), $I \neq \emptyset \subset \left[h_{i+1/2}, H_{i+1/2}\right]$

with $h_{i+1/2} = \min\left(\alpha_i, \alpha_{i+1}\right) = \min\left(\alpha_{i+1/2}^{up}, \alpha_{i+1/2}^{down}\right)$ and $H_{i+1/2} = \max\left(\alpha_i, \alpha_{i+1}\right)$

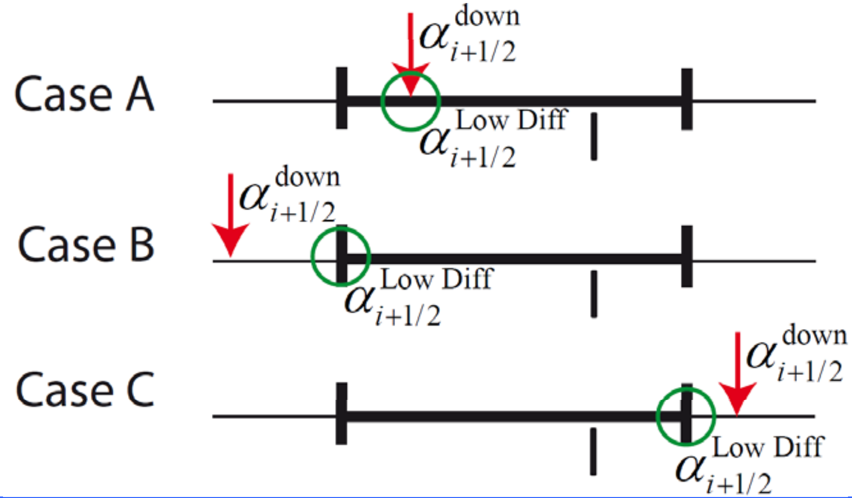
Maximum principle for $\nearrow c_{g\,i+1/2,j}^{n+1,L}$, $\searrow c_{g\,i,j}^{n+1,*}$ ⇒ If $\alpha_{i+1/2} \in I$, we ensure consistance and stability properties of the projection scheme for the concentration of gas c_g

$I = I_1 \cap I_2$ as an intersection of 2 conditions

2) Low-diffusive procedure:

$$\alpha_{i+1/2} = \alpha_{i+1/2}^{\text{LowDiff}}$$

Choose $\alpha_{i+1/2} \in I$ less diffusive as possible: take $\alpha_{i+1/2}^{\text{Low Diff}}$ the closest value to $\alpha_{i+1/2}^{\text{down}}$ but staying in I.



Low-diffusive projection

Theorem: We have construct a trust interval I which ensures good properties for our projection scheme:

Trust interval I:

$$\text{if} \left| \begin{array}{l} u_{i+1/2,j}^{n+1/2,L} > 0 \,\&\, u_{i-1/2,j}^{n+1/2,L} > 0 \\ \text{or} \\ u_{i+1/2,j}^{n+1/2,L} < 0 \,\&\, u_{i+3/2,j}^{n+1/2,L} < 0 \end{array} \right. , \quad I := \left[\omega_{i+1/2}, \Omega_{i+1/2}\right] = \underbrace{I_1}_{\text{consistance for } c_g} \cap \underbrace{I_2}_{\text{stability for } c_g}$$

ensuring a maximum principle for: the edges value (for I_1), the projected value (for I_2)

⇒ Under CFL condition $\Delta t \le 0.5 \dfrac{\min(\Delta x, \Delta y)}{\max(|u|,|v|,c)}$ and if $c > 2\max\left(|u|,|v|\right)$:

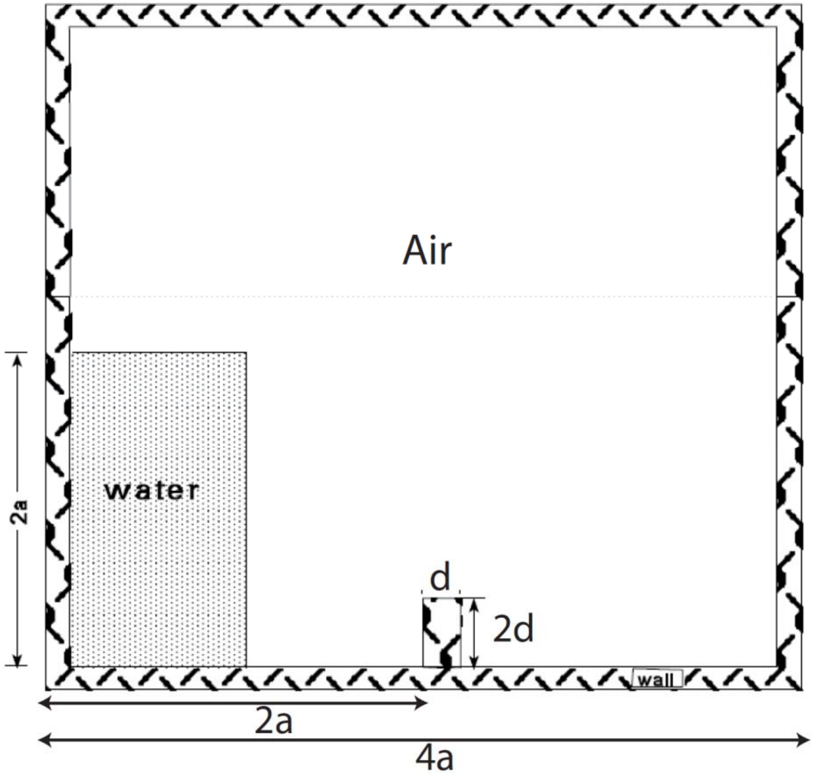
the trust interval $I = \left[\omega_{i+1/2}, \Omega_{i+1/2}\right] \neq \varnothing$ $\left(\alpha_{i+1/2,j}^{\text{upw}} = \begin{cases} \alpha_i^{n+1,L} & \text{if } u_{i+1/2,j}^{n+1/2,L} > 0 \\ \alpha_{i+1}^{n+1,L} & \text{else} \end{cases} \in I\right)$ and take

$\alpha_{i+1/2,j}^{\text{LowDiff}} \in I$ ensures the **positivity of masses** of each phases.

SIMULATIONS
WITH THE CODE ODYSSEY

Simulations

- Collapse of a liquid column with an obstacle



O. Ubbink Numerical prediction of two-fluid systems with sharp interfaces, PhD thesis (1997)

$$a = 0.146 \text{ m}, \; d = 0.024 \text{ m}$$

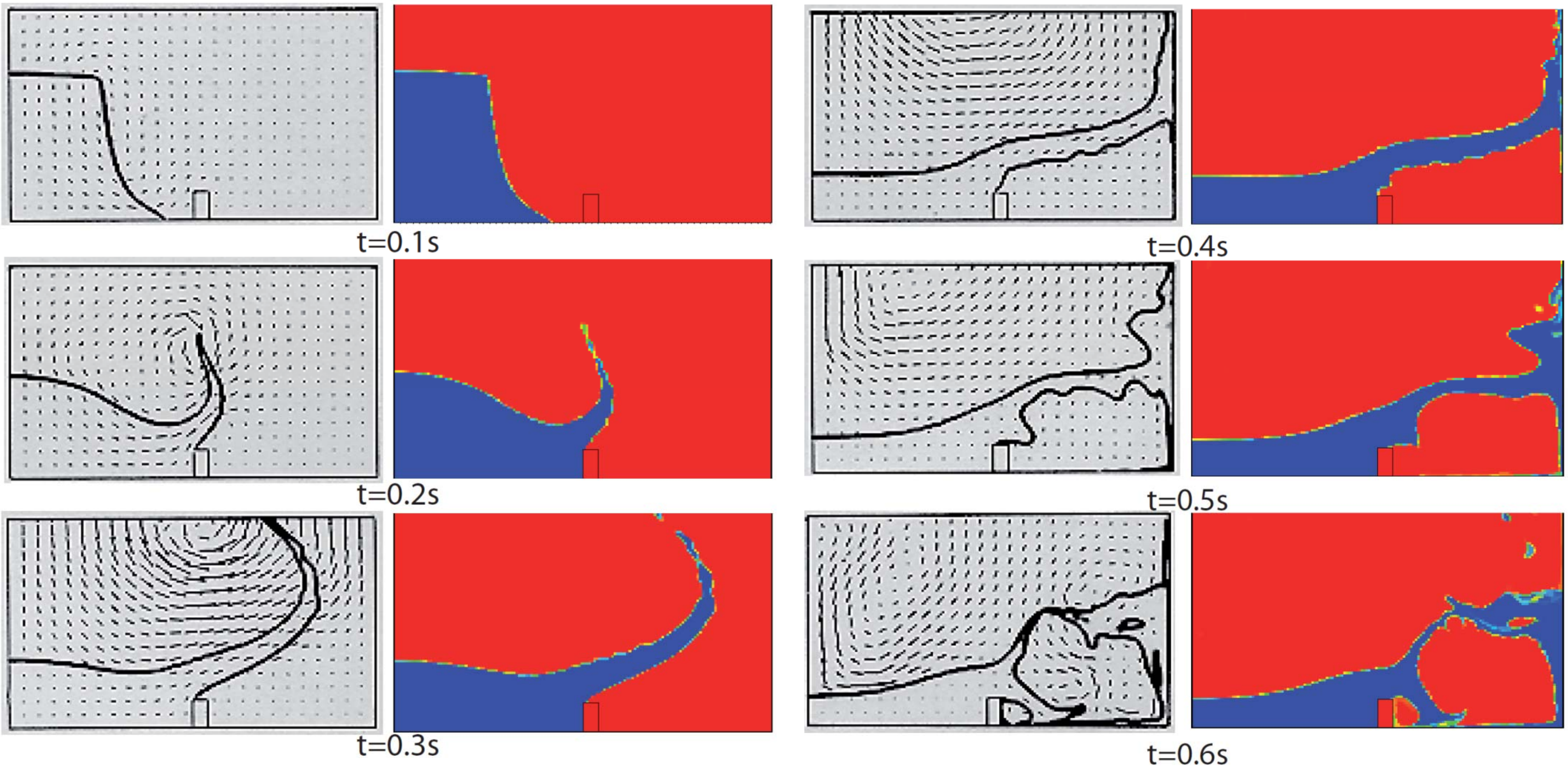
$$Nx = Ny = 150$$

$$\gamma_g = 1.4, \; \gamma_l = 7$$

$$\rho_0^g = 1.28 \text{ kg.m}^{-3}, \; \rho_0^l = 1000 \text{ kg.m}^{-3}$$

$$P_0 = 10^5, \; c_{\text{sound}} = 350 \text{ m.s}^{-1}$$

Simulations- Collapse with an obstacle

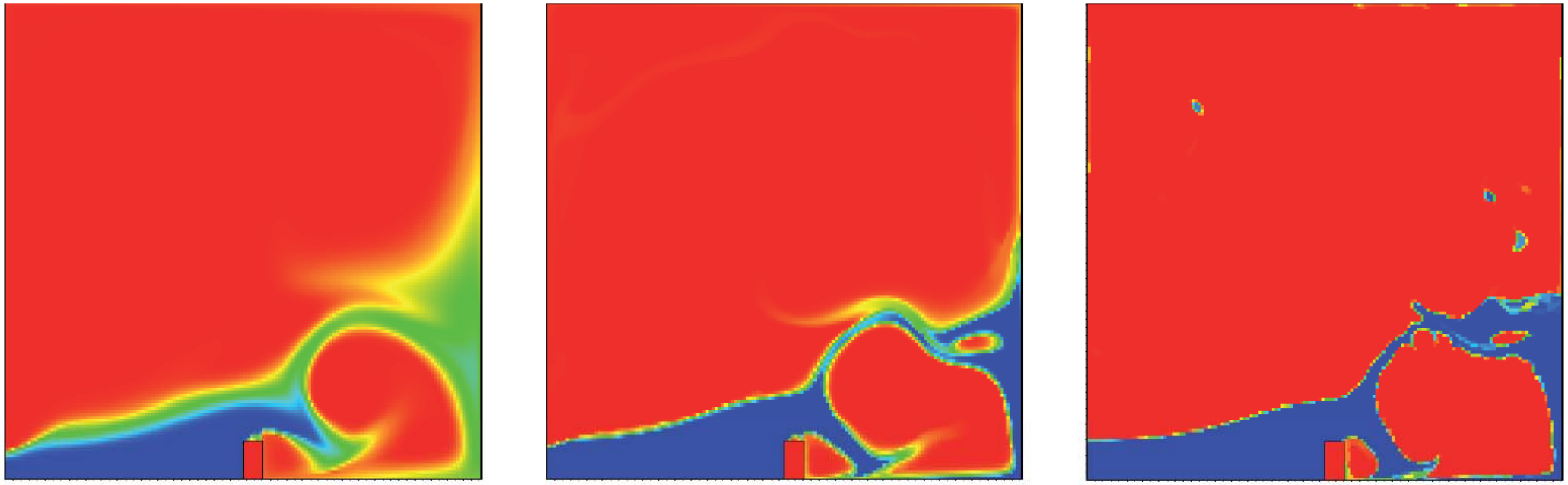


Num. Result. Ubbink | Num. Result. with Odyssey | Num. Result. Ubbink | Num. Result. with Odyssey

Simulations- Collapse with an obstacle

Evolution of α with the code Odyssey with diff.method of projection

t=0.6s

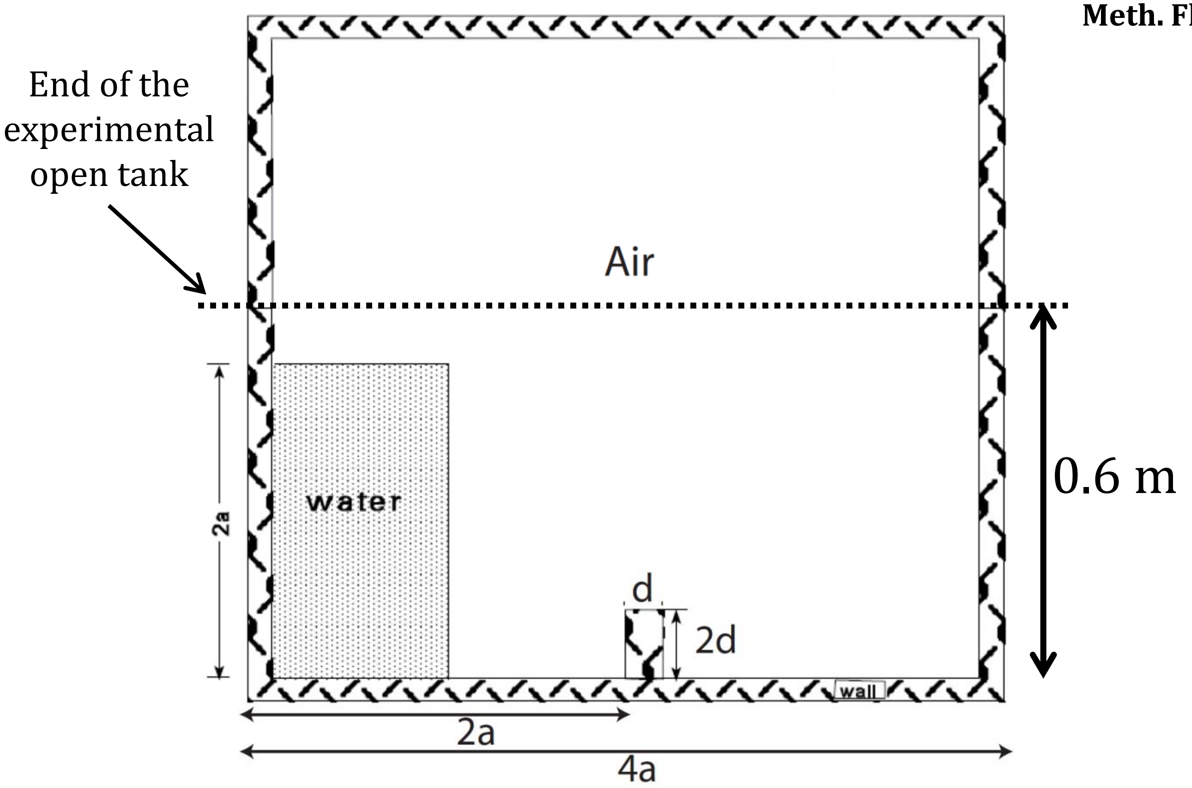


Projection: 1st order with upwind | 2nd order with upwind | 1st order with Low-Diff.

Simulations

- Collapse of a liquid column with an obstacle

D. M. Greeves Simulation of viscous water column collapse using adapting hierarchical grids J. Num. Meth. Fluids 2006



$$a = 0.25,\ d = 0.04$$

$$Lx = Ly = 1\ \text{m}$$

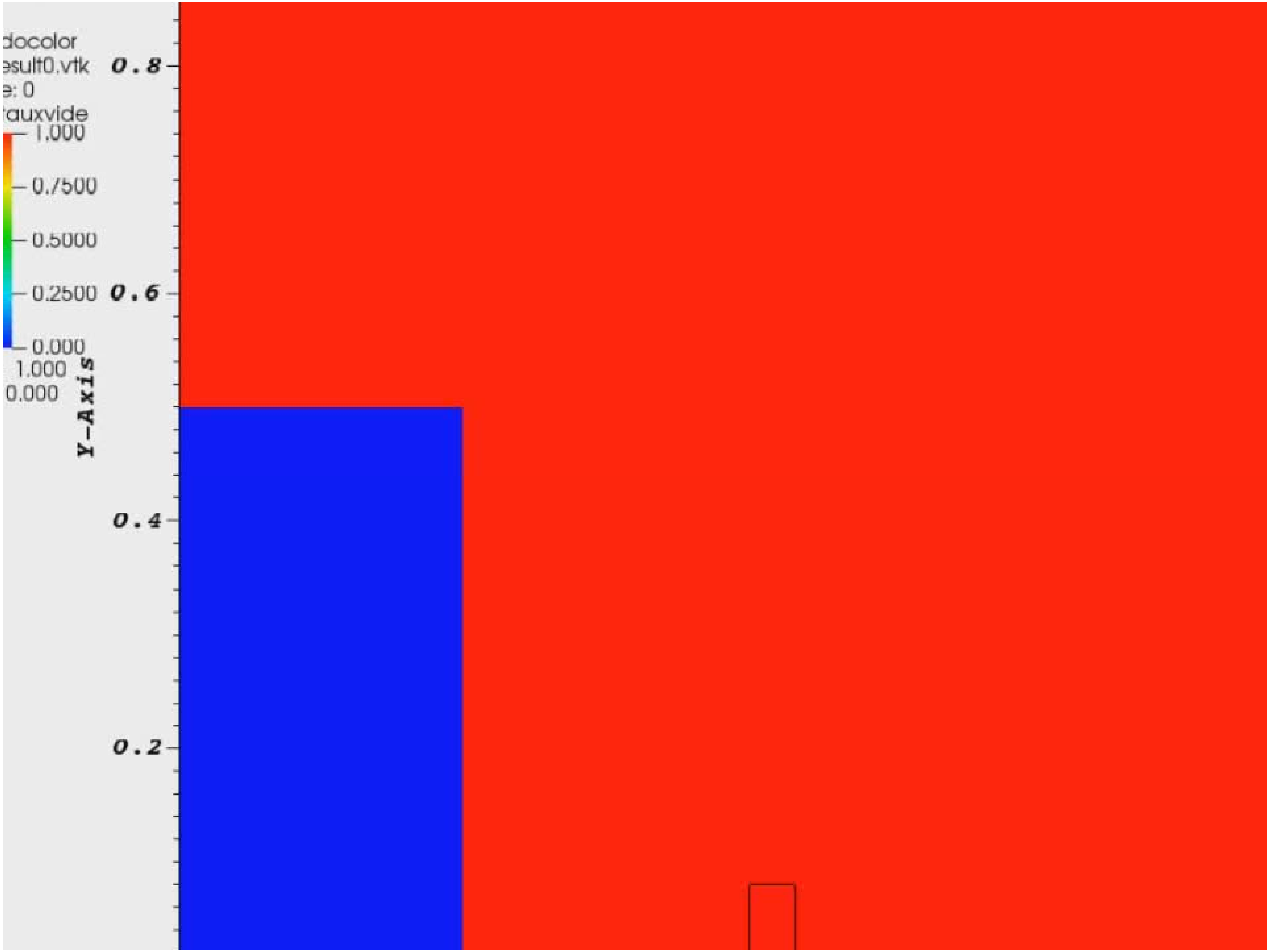
$$Nx = Ny = 600$$

$$\gamma_g = 1.4,\ \gamma_l = 7$$

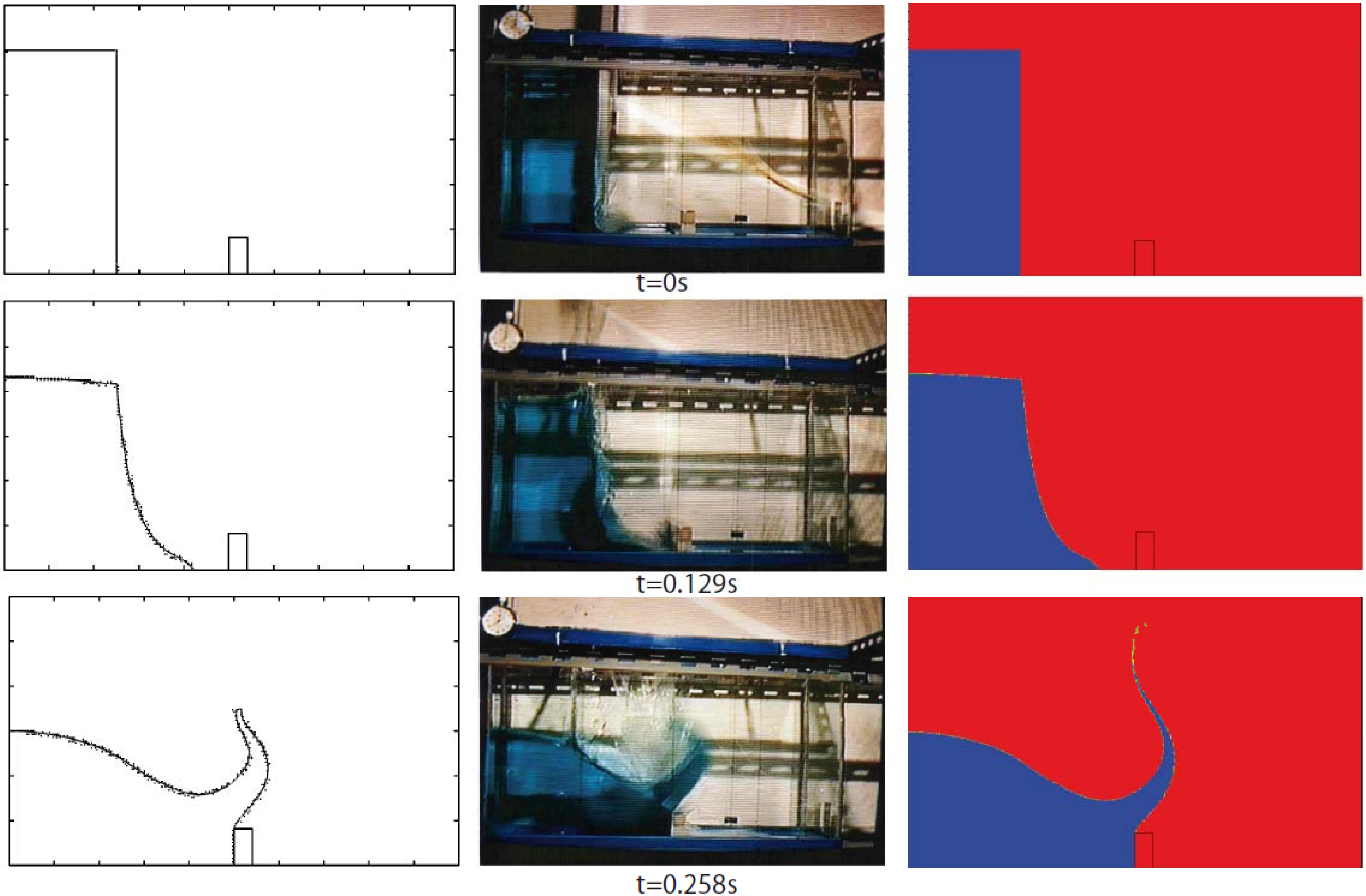
$$\rho_0^g = 1.\ \text{kg.m}^{-3},\ \rho_0^l = 1000\ \text{kg.m}^{-3}$$

$$P_0 = 10^5,\ c_{\text{sound}} = 350\ \text{m.s}^{-1}$$

Exp: from
Koshizuka et al A particle method.. Comp. Fluid Mech. 1995

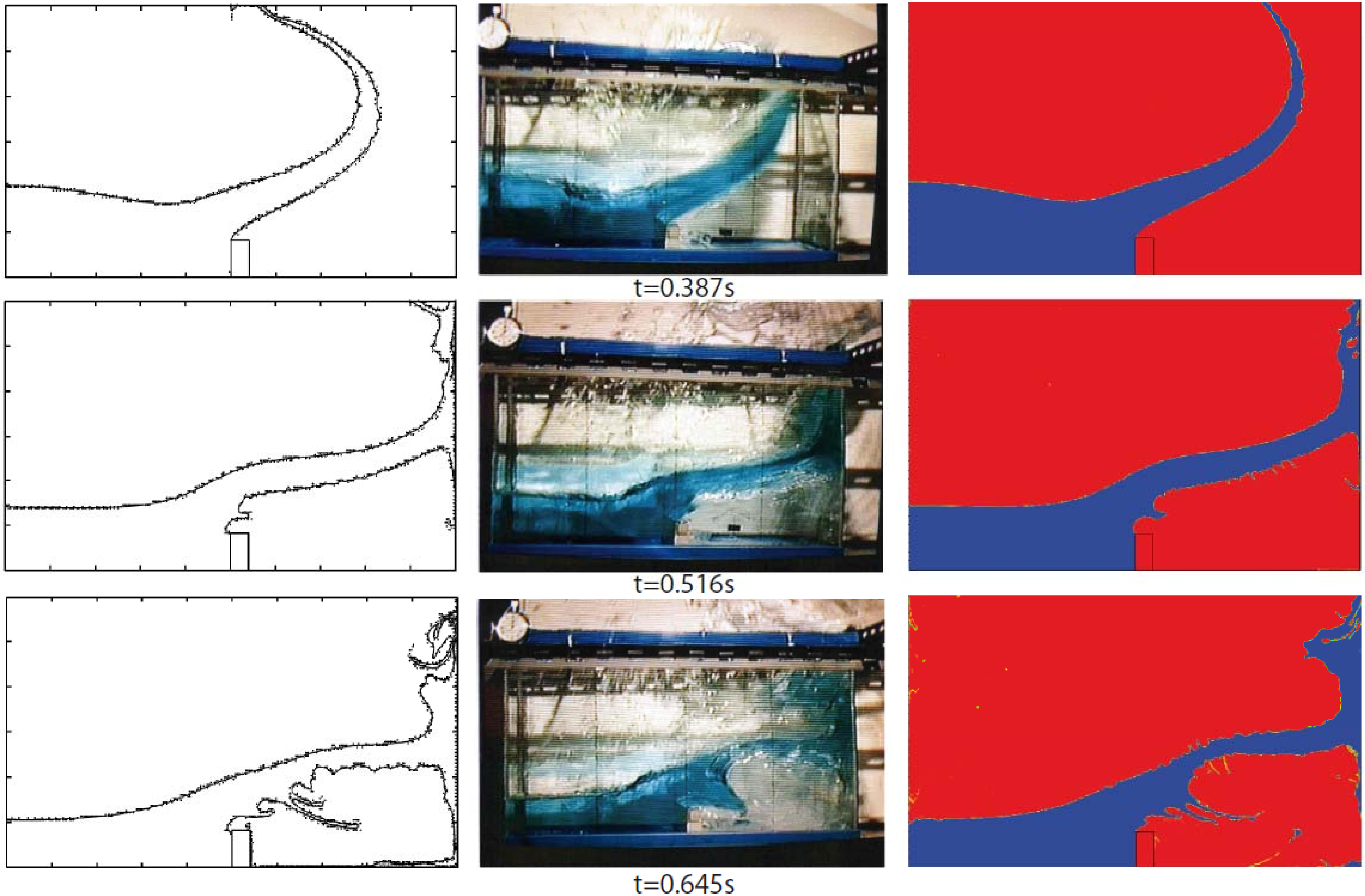

docolor
esult0.vtk
e: 0
auxvide
1.000
0.7500
0.5000
0.2500
0.000
1.000
0.000
Y-Axis
0.8
0.6
0.4
0.2

Simulations of a Dam-break



Sim. Greeves Exp. Koshizuka Sim. Odyssey

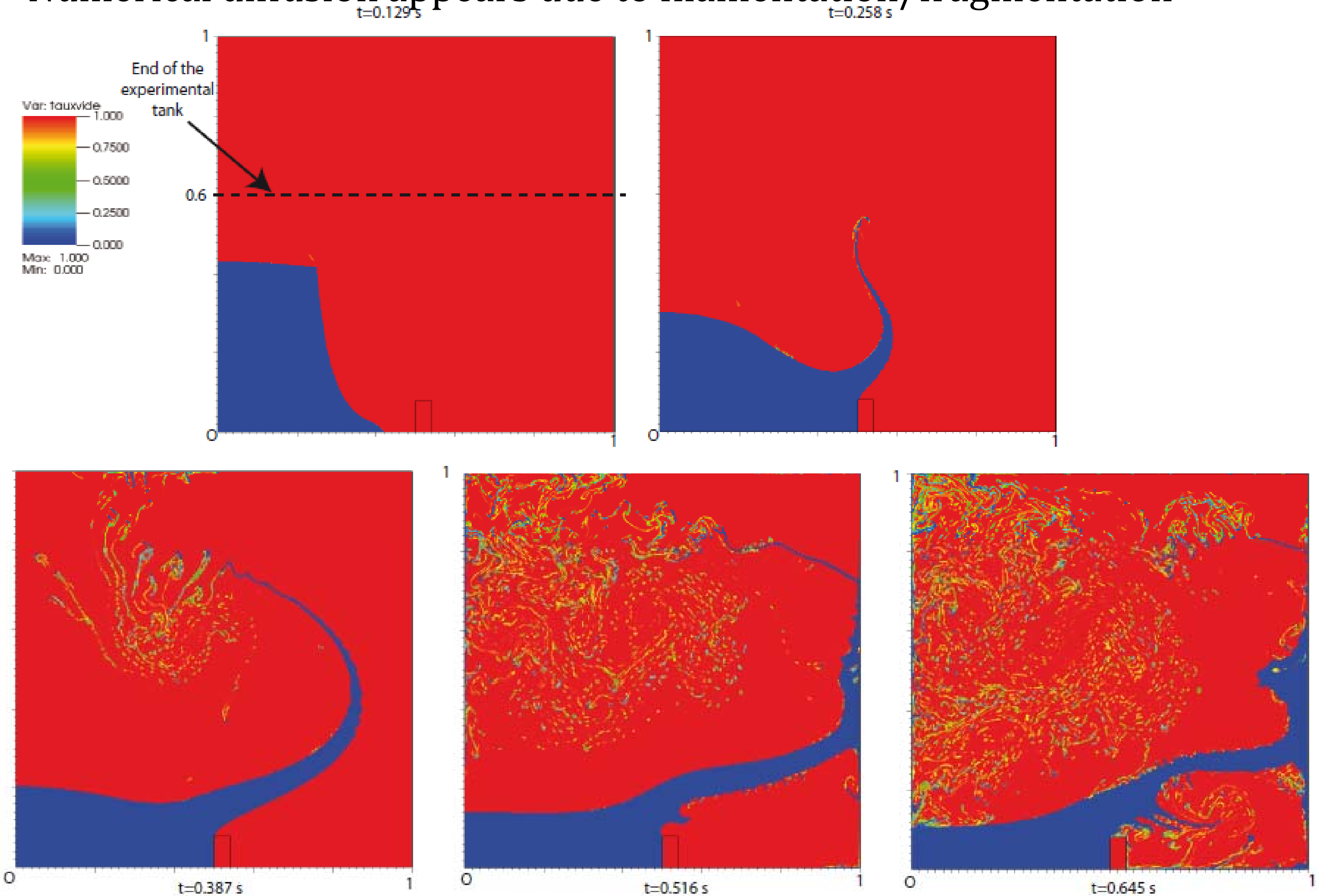
Simulations of a Dam-break



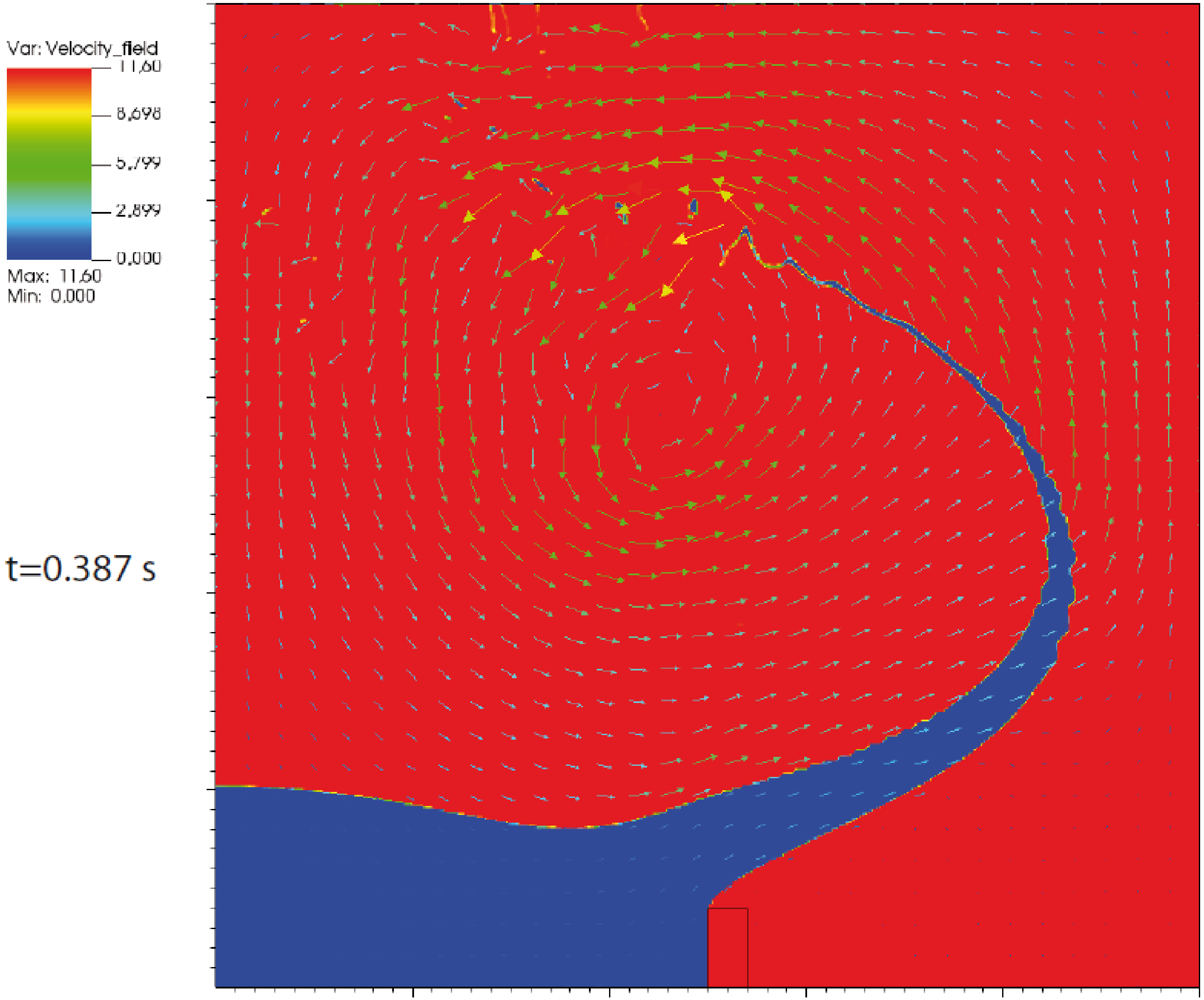
Sim. Greeves Exp. Koshizuka Sim. Odyssey

Gas mass fraction

- Numerical diffusion appears due to filamentation/fragmentation

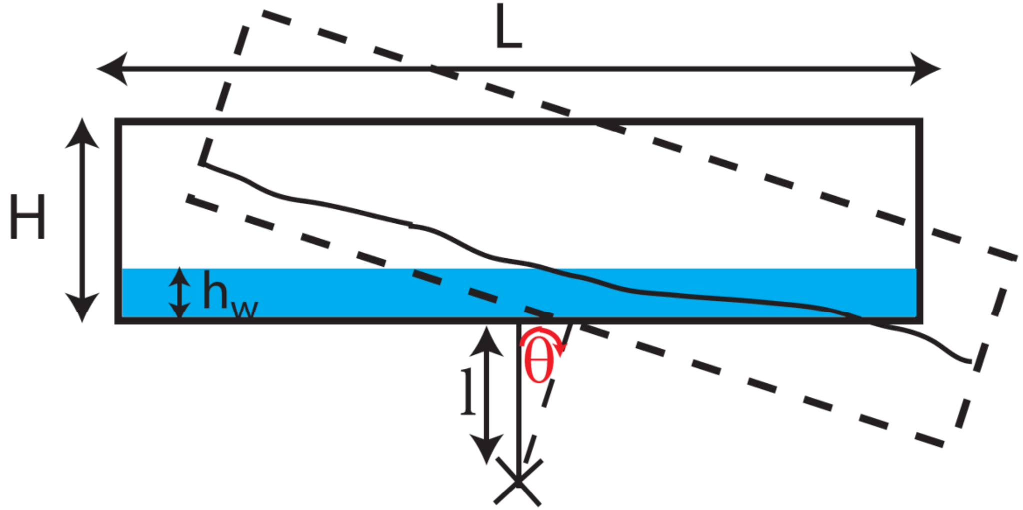


Velocity field



Sloshing test cases – pitch motion

- Sloshing due to the **pitch motion** of a rectangular tank:



J.R. Shao *et al* . An improved SPH method for modeling liquid sloshing dynamics. Comp. Fluids 2012

$$L = 0.64m,\ H = 0.14m,\ h_w = 0.03m$$
$$Nx = 300,\ Ny = 67$$

The tank is oscillating as a **pendulum** according to:

$$\theta(t) = \theta_0 \sin(\omega_r t)$$

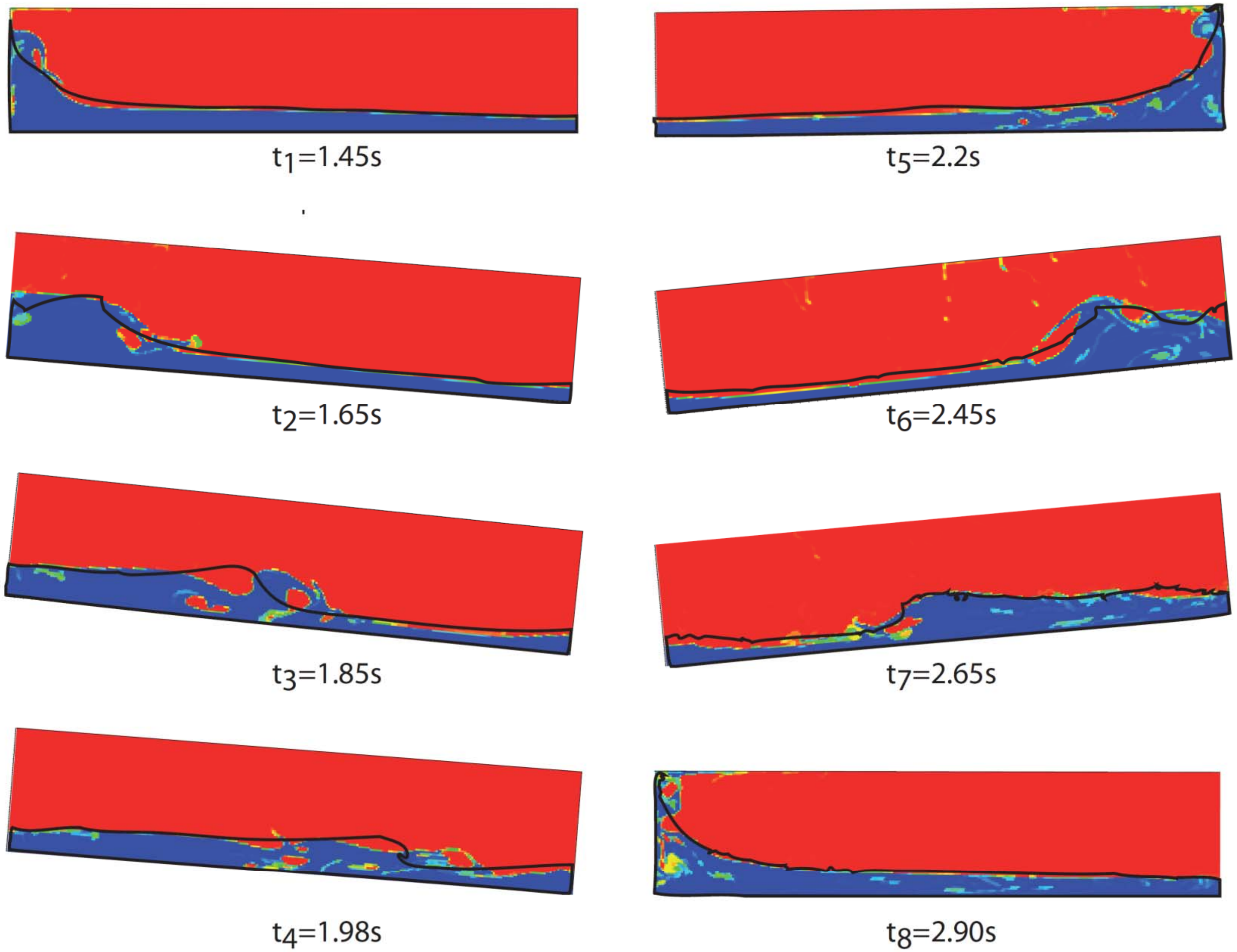
with $\theta_0 = 6^\circ,\ \omega_r = 4.34 \text{ rad/s } (T = 1.45 \text{ s})$

⇒ Simulation are performed in the frame of reference of the tank.

Sloshing test cases – pitch motion

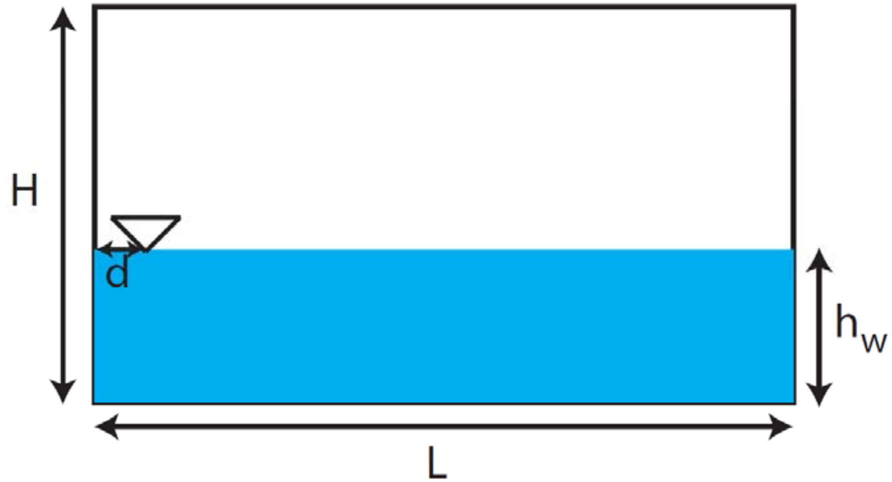
Sloshing test cases – pitch motion

We superimpose the profile of the article: **J.R. Shao *et al* . An improved SPH method for modeling liquid sloshing dynamics. Comp. Fluids 2012**



Sloshing test cases – surge motion

- Sloshing due to the **surge motion** of a rectangular tank:



J.R. Shao *et al* . An improved SPH method for modeling liquid sloshing dynamics. Comp. Fluids 2012

$$L = 1.73m,\ H = 1.15m,\ h_w = 0.6m$$
$$d = 0.05 \text{ m}$$
$$Nx = 173,\ Ny = 115$$

The tank is moving horizontally according to:

$$x(t) = A\cos\left(\frac{2\Pi t}{T}\right)$$

with $A = 0.032$ m, $T = 1.3$ s (ω_{forced} =4.83 rad/s).

First natural frequency of the fluid in the box

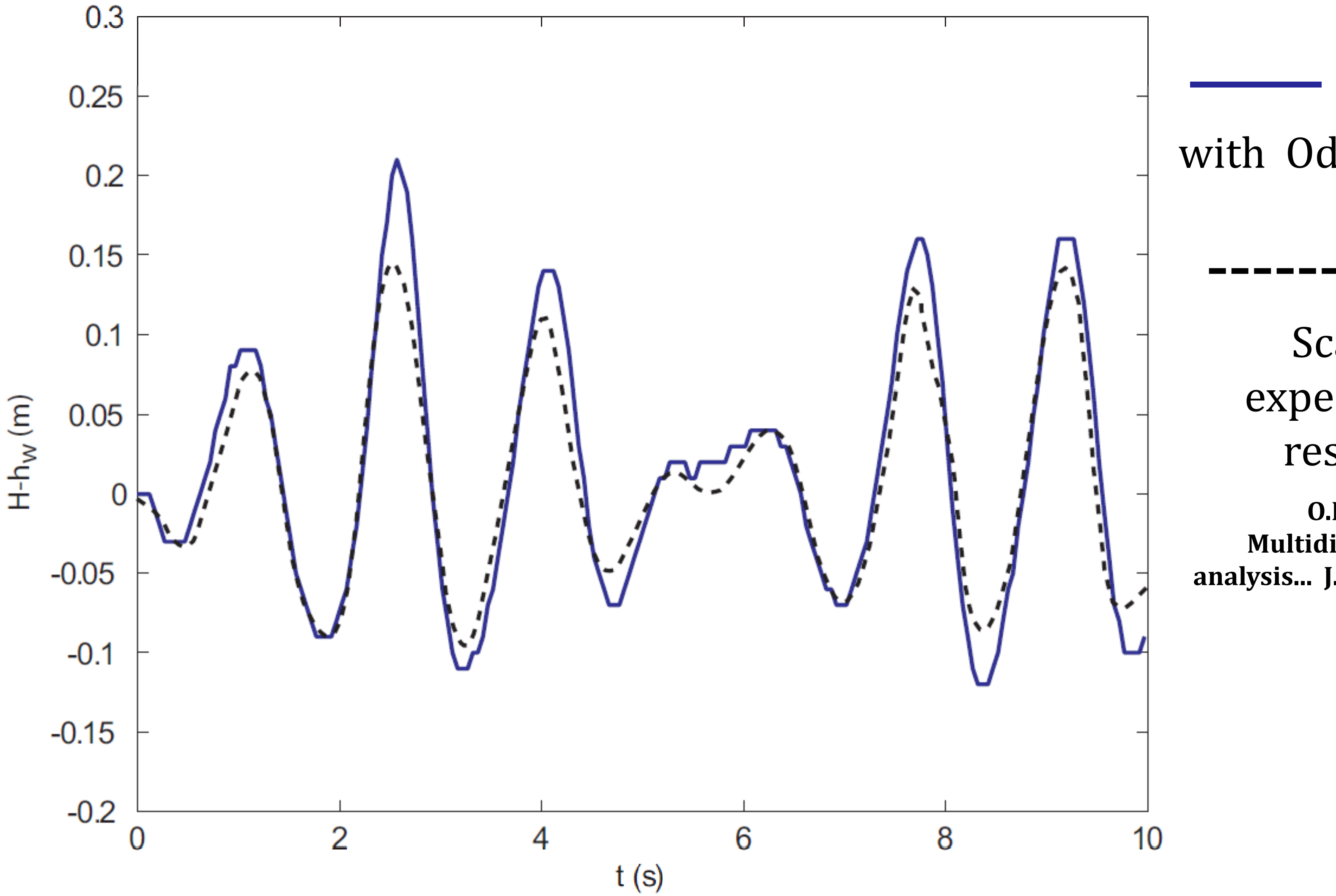
$$\omega_{\text{fluid}} = \sqrt{g\frac{\pi}{L}\tanh(\frac{\pi}{L}h_w)} \approx 3.77 \text{ rad/s}$$

Two frequencies are acting ω_{fluid} and ω_{forced}

Experimentals results are available: **O.M. Faltinsen *et al* . Multidimensional modal analysis... J. Fluid. Mechanics 2000**

Sloshing test cases – surge motion

Free surface elevation of water at the probe



with Odyssey

Scanned experimental results of

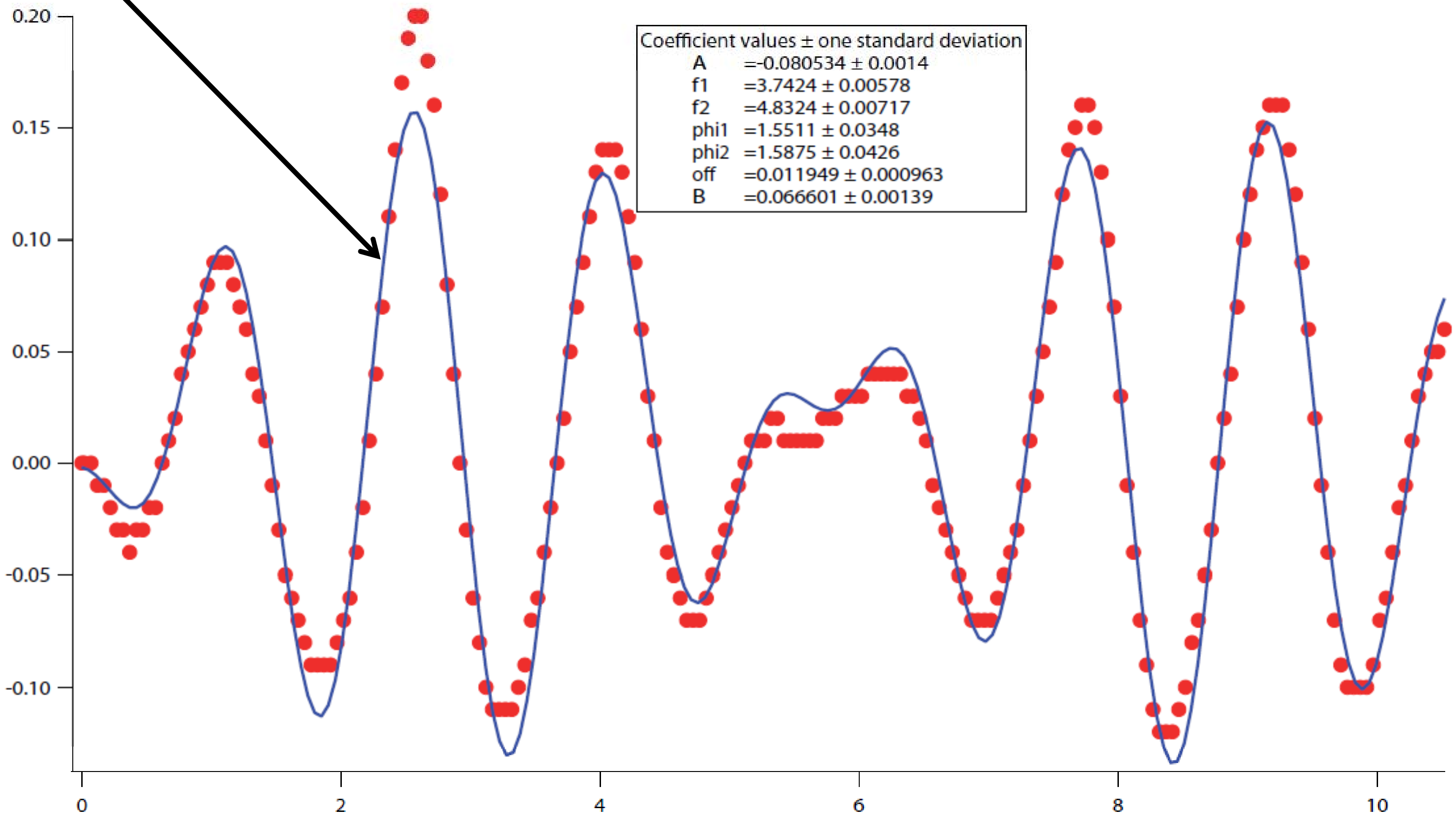
O.M. Faltinsen *et al* . Multidimensional modal analysis... J. Fluid. Mechanics 2000

Sloshing test cases – surge motion

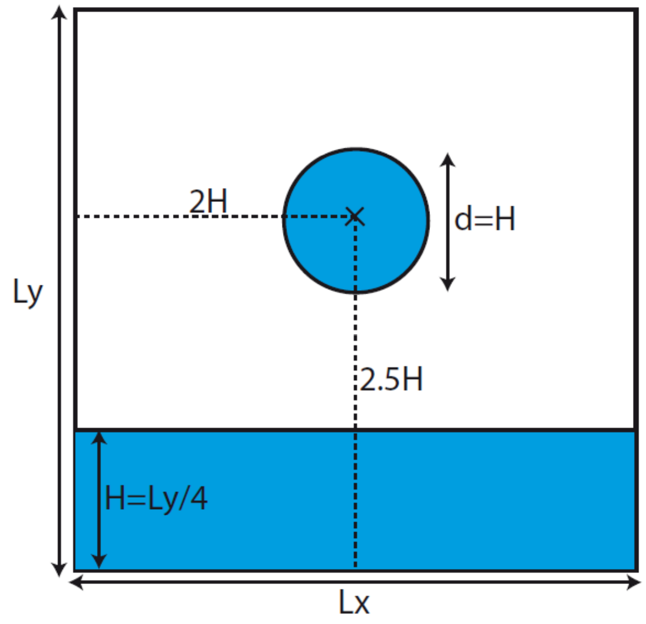
We search to **find a fit of** our curve with a function as a superposition of two signals

$f(t) = A_1 \sin(\omega_1 t + \varphi_1) + A_2 \sin(\omega_2 t + \varphi_2)$, we get: $\omega_1 = 3.74 \pm 0.01$ rad/s, $\omega_2 = 4.83 \pm 0.01$ rad/s

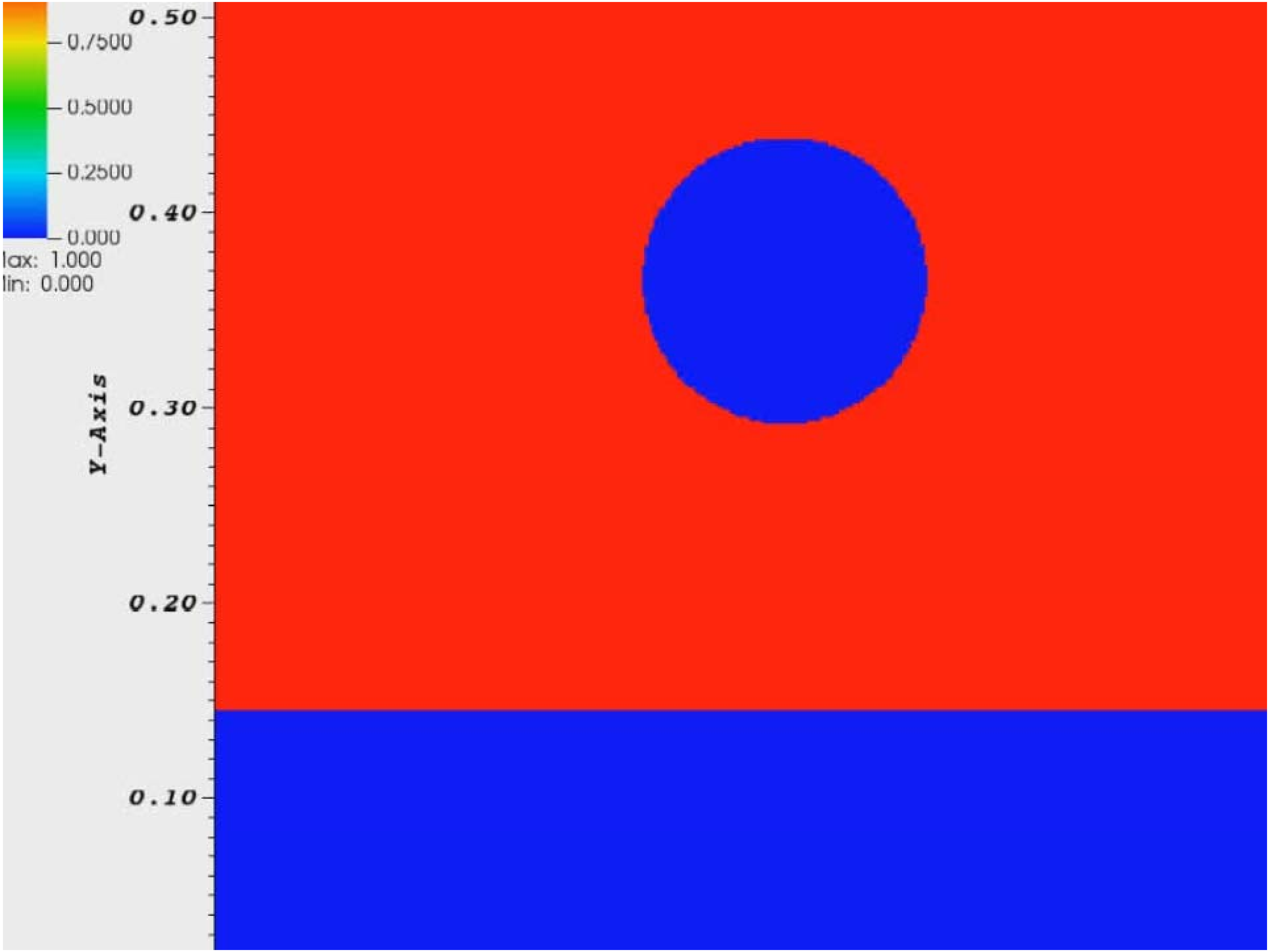
very close to $\omega_{\text{fluid}} \approx 3.77$ rad/s and $\omega_{forced} = 4.83$ rad/s



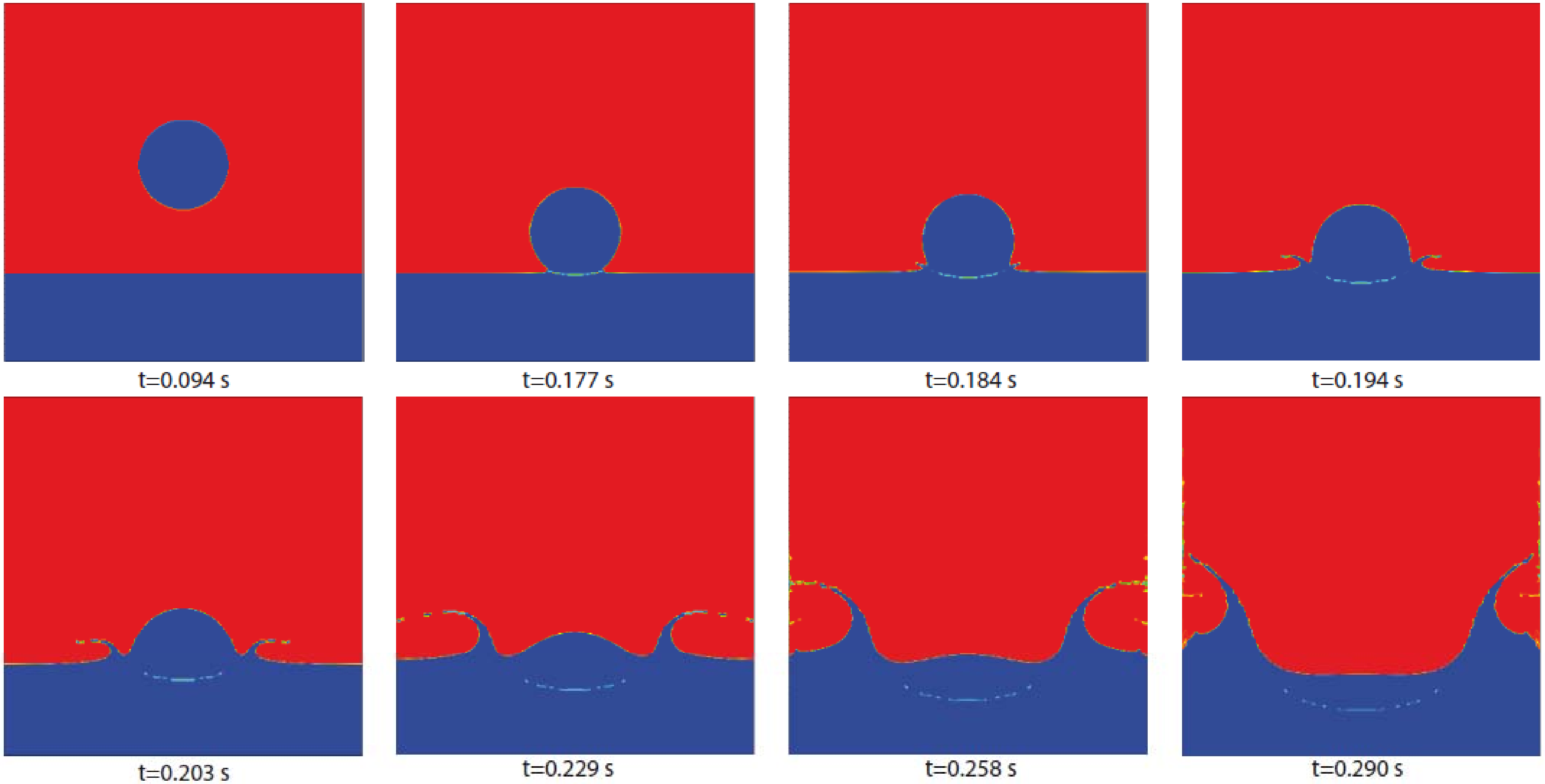
Free fall of liquid and impact with liquid at rest



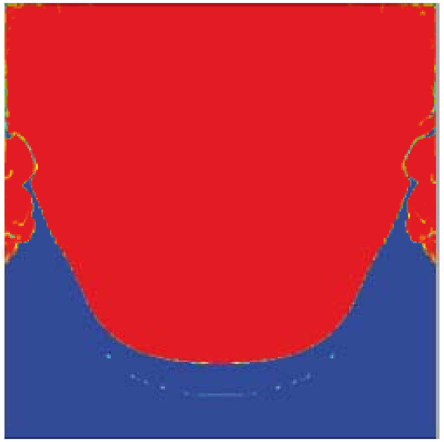
$$Lx = Ly = 0.584 \text{ m}$$
$$H = Ly / 4 = 0.146 \text{ m}$$
$$d = H = 0.146 \text{ m}$$
$$Nx = Ny = 350$$

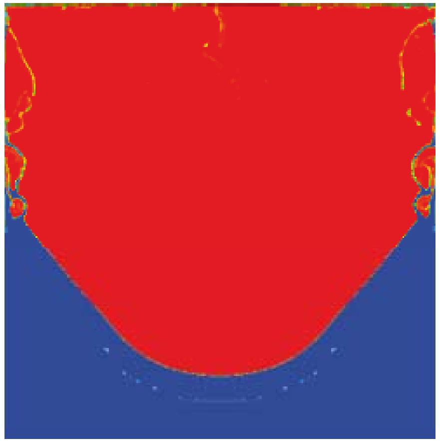

0.7500
0.5000
0.2500
0.000
lax: 1.000
lin: 0.000
Y-Axis
0.50
0.40
0.30
0.20
0.10

Free fall of liquid and impact with liquid at rest



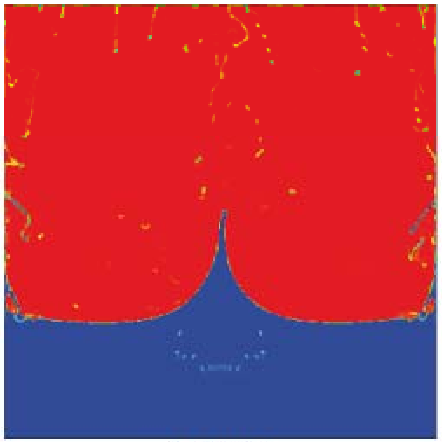
Free fall of liquid and impact with liquid at rest

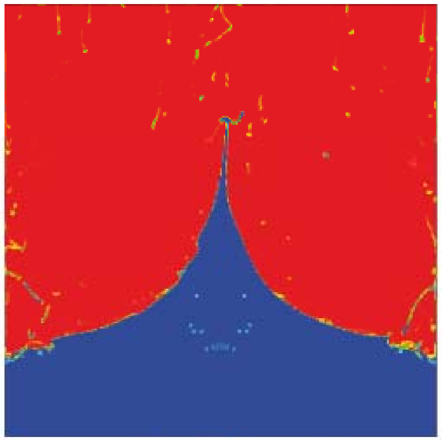

t=0.349 s

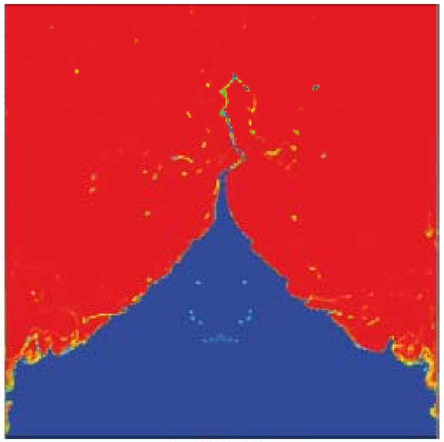

t=0.450 s

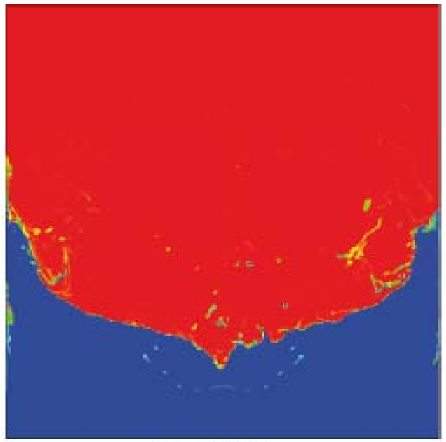

t=0.545 s


t=0.564 s


t=0.618 s


t=0.694 s


t=0.859 s


t=1.055 s

Conclusion

- Simulations of free-surface flows with an Eulerian numerical method
  - Lagrange-Remap scheme
  - An anti-diffusive approach on the gas mass fraction

- The low-diffusive projection allows us to keep a thin interface between the two-fluids
  - We can observe the formation of air-pocket, formation of blast, ejection of droplets, fragmentation…
- The test cases show a good agreement between experiences and other codes : dam break, sloshing events.

- Some videos of simulation with the code Odyssey are available on You Tube:

  http://www.youtube.com/user/audeBChampmartin

  http://www.youtube.com/user/floriandevuyst

Perspectives

- Allowing more boundary conditions in the code (only wall boundary condition up to now)
- Taking into account physical viscosity, surface tension.
- Parallelization of the code.

- Publications
  - "A low diffusive Lagrange-remap scheme for the simulation of violent air water free-surface flows." A.Bernard-Champmartin and F. De Vuyst.

    submitted to Journal of Computational Physics

    http://hal.archives-ouvertes.fr/hal-00798783
  - "A low diffusive Lagrange-remap scheme for the simulation of violent air water free-surface flows. Applications to dam-break and sloshing events." In preparation.