Numerical simulation of depth averaged flow models : a class of Finite Volume and discontinuous Galerkin approaches.

Arnaud Duran
GdR EGRIN, 1^{er} juin 2015

INSA Toulouse,
INRIA, Team LEMON.

2D Formulation

$$\frac{\partial}{\partial t}U + \nabla.G(U) = B(U,z) + F(U)\,.$$

$$U = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix}, \quad G(U) = \begin{pmatrix} hu & hv \\ gh^2 + hu^2 & huv \\ huv & gh^2 + hv^2 \end{pmatrix},$$

$$\text{Source terms}: B(U,z) = \begin{pmatrix} 0 \\ -gh\partial_x z \\ -gh\partial_y z \end{pmatrix}, \quad F(U) = \begin{pmatrix} 0 \\ -n^2 \frac{\|\mathbf{q}\|}{h^{10/3}} hu \\ -n^2 \frac{\|\mathbf{q}\|}{h^{10/3}} hv \end{pmatrix}.$$

Application field : some examples

Coastal hydrodynamic

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FV and dG schemes for SW et GN Eq.
Arnaud Duran
Shallow Water Equations
Introduction
Numerical issues
Treatment of the bottom
Extension to dispersive equations
Perspectives

Shallow Water Equations

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Application field : some examples

Tsunamis

2D Formulation

$$\frac{\partial}{\partial t}U + \nabla.G(U) = B(U,z) + F(U).$$

$$U = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix}, \quad G(U) = \begin{pmatrix} hu & hv \\ gh^2 + hu^2 & huv \\ huv & gh^2 + hv^2 \end{pmatrix},$$

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Application field : some examples

Rivers, dam breaks

Stability criteria

- Preservation of the “lake at rest” :

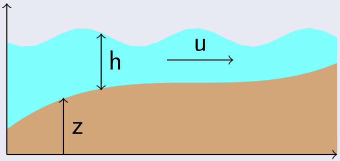
$$h + z = cte\,,\; \mathbf{u} = 0\,.$$

- Robustness : preservation of the water depth positivity.
- Entropy inequalities.

Outline

Outline

1D Configuration



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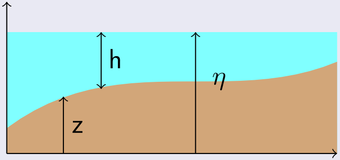
"Pre-Balanced" formulation.
- [Q. Liang and F. Marche. Numerical resolution of well-balanced shallow water equations with complex source terms. Adv. Water Res., 2009.]
1D Finite Volume scheme.

1D Lake at rest configuration



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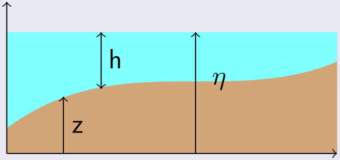

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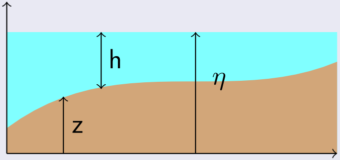

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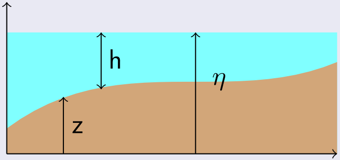


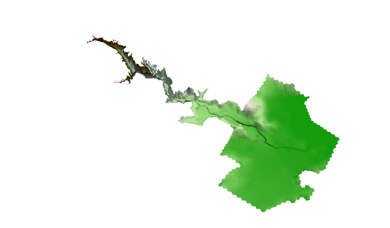
Pre balanced formulation

$$\frac{\partial}{\partial t}V + \nabla.H(V,z) = S(V,z)\,.$$

$$V = \begin{pmatrix} \eta \\ hu \\ hv \end{pmatrix}\ ,\ H(V,z) = \begin{pmatrix} hu & hv \\ \frac{1}{2}g(\eta^2 - 2\eta z) + hu^2 & huv \\ huv & \frac{1}{2}g(\eta^2 - 2\eta z) + hv^2 \end{pmatrix}\,.$$

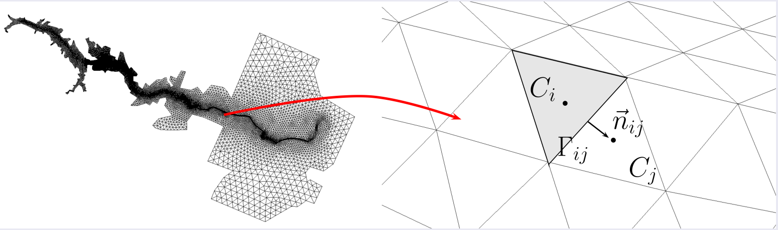
Source term : $\quad S(V,z) = \begin{pmatrix} 0 \\ -g\eta\partial_x z \\ -g\eta\partial_y z \end{pmatrix}\,.$

Geometry and unstructured meshes



Finite Volume formulation

Geometry and unstructured meshes



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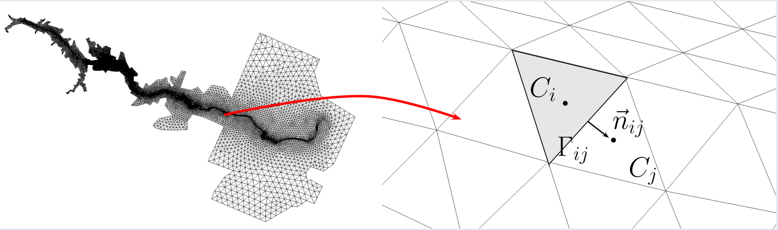
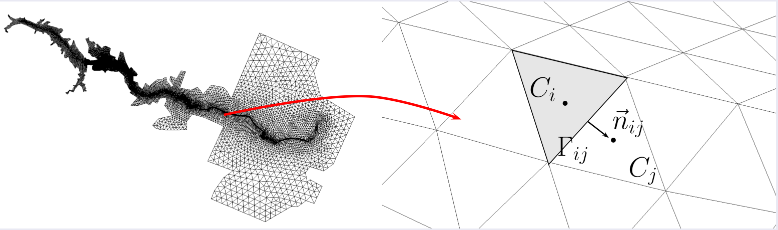


Finite Volume formulation

$$\frac{\partial}{\partial t}V + \nabla.H(V,z) = S(V,z)\,.$$

Geometry and unstructured meshes

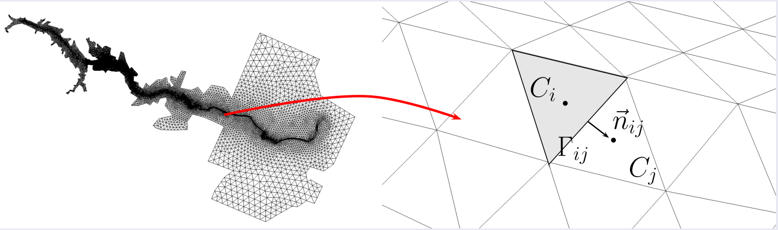


Finite Volume formulation

$$\int_{C_i} \frac{\partial}{\partial t} V \, d\mathbf{x} + \int_{C_i} \nabla . H(V, z) \, ds = \int_{C_i} S(V, z) d\mathbf{x} \, .$$

Generalities

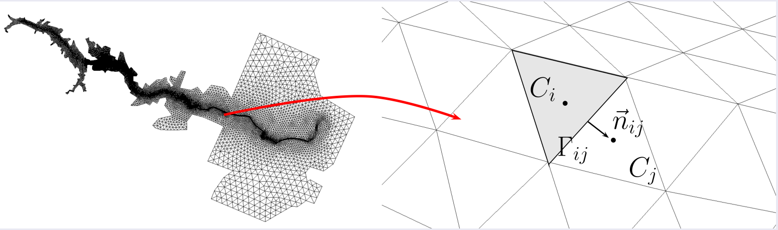
Geometry and unstructured meshes



Finite Volume formulation

$$\int_{C_i} \frac{\partial}{\partial t} V \, d\mathbf{x} + \int_{\partial C_i} H(V,z).\vec{n} \, ds = \int_{C_i} S(V,z) d\mathbf{x} .$$

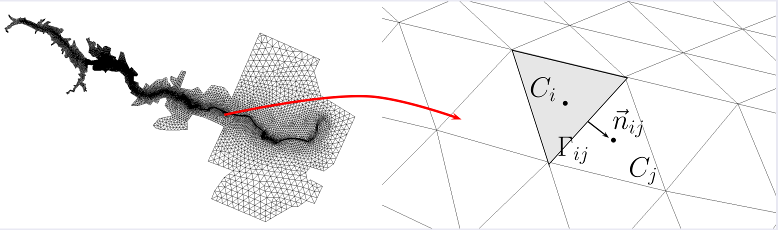
Geometry and unstructured meshes



Finite Volume formulation

$$\int_{C_i} \frac{\partial}{\partial t} V\, d\mathbf{x} + \sum_{k=1}^{\Lambda(i)} \int_{\Gamma_{ij(k)}} H(V,z).\vec{n}_{ij(k)}\, ds = \int_{C_i} S(V,z) d\mathbf{x}.$$

Geometry and unstructured meshes

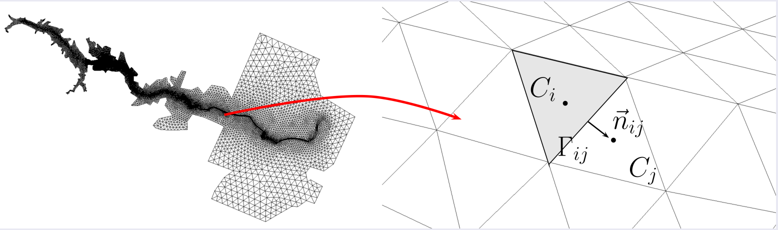


Finite Volume formulation

$$|C_i|\frac{d}{dt}V_i + \sum_{k=1}^{\Lambda(i)} \int_{\Gamma_{ij(k)}} H(V,z).\vec{n}_{ij(k)}\, ds = \int_{C_i} S(V,z)d\mathbf{x}\,,$$

$$V_i = \frac{1}{|C_i|}\int_{C_i} V(\mathbf{x},t)d\mathbf{x}\,.$$

Geometry and unstructured meshes



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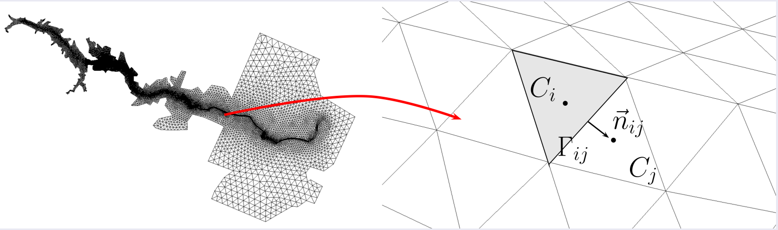
Numerical fluxes : $\displaystyle\int_{\Gamma_{ij(k)}} H(V,z).\vec{n}_{ij(k)}\, ds \approx \ell_{ij(k)}\mathcal{H}_{ij(k)}\,.$

Geometry and unstructured meshes



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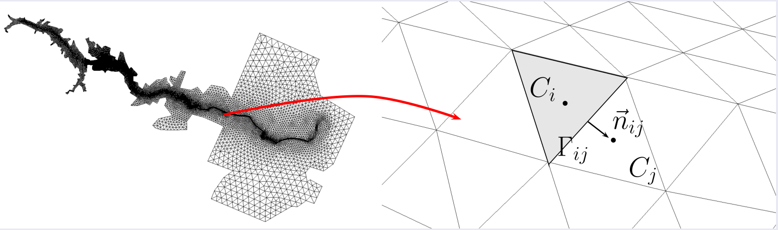
$$|C_i|\frac{d}{dt}V_i + \sum_{k=1}^{\Lambda(i)} \underbrace{\ell_{ij(k)}\mathcal{H}_{ij(k)}}_{\text{Numerical fluxes}} = S_i\,.$$

Geometry and unstructured meshes



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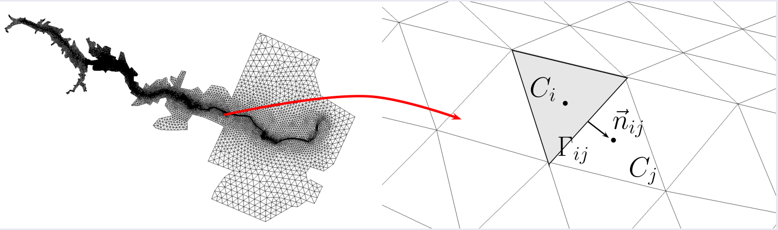

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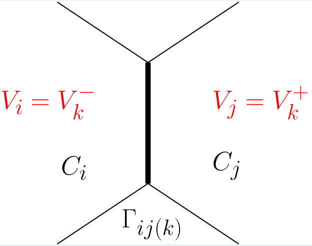
Reconstruction at the interfaces



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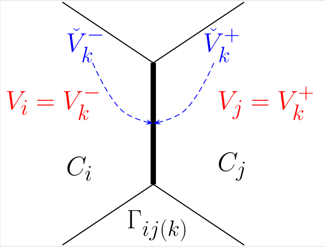

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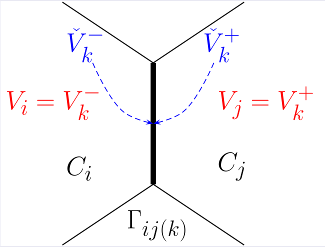
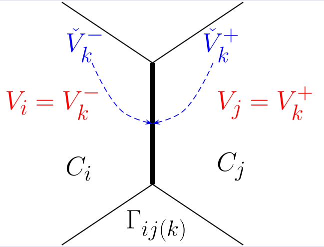


$$\tilde{z}_k = \max(z_k^-, z_k^+),$$

Evaluation of the numerical fluxes

Reconstruction at the interfaces



$$\tilde{z}_k = \max(z_k^-, z_k^+)\,,$$

$$\check{h}_k^- = \max(0, \eta_k^- - \tilde{z}_k)\,,$$

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Evaluation of the numerical fluxes

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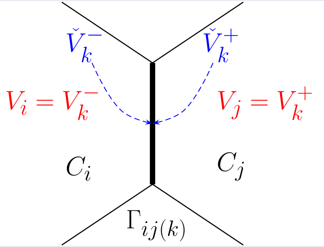

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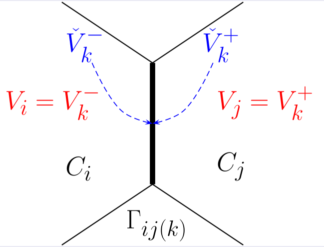
Reconstruction at the interfaces



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$$\check{\eta}_k^- = \check{h}_k^- + \check{z}_k \quad , \quad \check{\mathbf{q}}_k^- = \frac{\check{h}_k^-}{h_k^-}\mathbf{q}_k^-\,,$$
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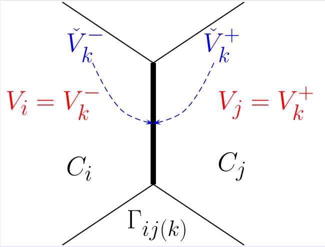

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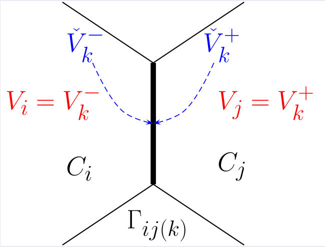
[**E. Audusse et al**. A fast and stable well-balanced scheme with hydrostatic reconstruction for shallow water flows. SIAM J. Sci. Comput., 2004.]

Reconstruction at the interfaces



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General formulation

$$\mathcal{H}_{ij(k)} = \mathcal{H}(\check{V}_k^-, \check{V}_k^+, \check{z}_k, \check{z}_k, \vec{n}_{ij(k)})\,.$$

Numerical scheme :

$$V_i^{n+1} = V_i^n - \frac{\Delta t}{|C_i|} \sum_{k=1}^{\Lambda(i)} \ell_{ij(k)} \mathcal{H}_{ij(k)} + \Delta t S_i \,. \qquad (1)$$

Objective : Preservation of the motionless steady states.
Key idea : Involve the properties of the 1D scheme.

Approximation of the source term

$$S_i = -\frac{1}{|C_i|} \sum_{k=1}^{\Lambda(i)} \ell_{ij(k)} S_{ij(k)} = -\frac{1}{|C_i|} \sum_{k=1}^{\Lambda(i)} \ell_{ij(k)} \begin{pmatrix} 0 \\ g\hat{\eta}_k (z_i - \check{z}_k) \vec{n}_{ij(k)} \end{pmatrix} .$$

→ Provided an appropriate choice of $\hat{\eta}_k$, the scheme (1) can be rewritten as a convex combination of 1D schemes.

The scheme written under convex combination

$$V_i^{n+1} = \sum_{k=1}^{\Lambda(i)} \frac{|T_{ij(k)}|}{|C_i|} V_{ij(k)}^{n+1} ,$$

$$V_{ij(k)}^{n+1} = V_i^n - \frac{\Delta t}{\Delta_{ij(k)}} \Big(\mathcal{H}_{ij(k)} - H(V_i, z_i).\vec{n}_{ij(k)} \Big) + \frac{\Delta t}{\Delta_{ij(k)}} S_{ij(k)} .$$

- Exact preservation of “lake at rest" configurations.
- Preservation of the positivity of the water depth.
- Stability and robust treatment in the neighbourhood of dry areas and wet/dry interfaces.

The scheme written under convex combination

$$V_i^{n+1} = \sum_{k=1}^{\Lambda(i)} \frac{|T_{ij(k)}|}{|C_i|} V_{ij(k)}^{n+1},$$

$$V_{ij(k)}^{n+1} = V_i^n - \frac{\Delta t}{\Delta_{ij(k)}}\Big(\mathcal{H}_{ij(k)} - H(V_i, z_i).\vec{n}_{ij(k)}\Big) + \frac{\Delta t}{\Delta_{ij(k)}} S_{ij(k)}\,.$$

- Exact preservation of “lake at rest" configurations.
- Preservation of the positivity of the water depth.
- Stability and robust treatment in the neighbourhood of dry areas and wet/dry interfaces.

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Numerical fluxes

$$\mathcal{H}_{ij(k)} = \mathcal{H}(\check{V}_k^-, \check{V}_k^+, \check{z}_k, \check{z}_k, \vec{n}_{ij(k)})\,.$$

Stability

- Well - balancing.
- Robustness.

Numerical fluxes

$$\mathcal{H}_{ij(k)} = \mathcal{H}\left(\breve{V}_k^{*,-}, \breve{V}_k^{*,+}, \breve{z}_k^{*}, \breve{z}_k^{*}, \vec{n}_{ij(k)}\right).$$

[**S. Camarri et al**. A low-diffusion MUSCL scheme for LES on unstructured grids. Comput. & Fluids, 2004.]

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- Well - balancing.
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[**S. Camarri et al**. A low-diffusion MUSCL scheme for LES on unstructured grids. Comput. & Fluids, 2004.]

Stability

- Well - balancing.
- Robustness.

MUSCL extension

Numerical fluxes

$$\mathcal{H}_{ij(k)} = \mathcal{H}\left(\check{V}_k^{*,-}, \check{V}_k^{*,+}, \check{z}_k^{*}, \check{z}_k^{*}, \vec{n}_{ij(k)} \right).$$

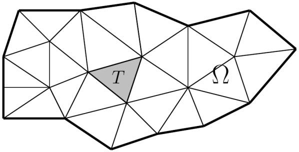
[**S. Camarri et al**. A low-diffusion MUSCL scheme for LES on unstructured grids. Comput. & Fluids, 2004.]

Stability

- Well - balancing.
- Robustness.
  [**C. Berthon**. Robustness of MUSCL schemes for 2D unstructured meshes. J. Comput. Phys., 2006.]

$\mathbb{P}^d(T) := \{$2 variables polynomials on T of degree at most $d\}$.

$\mathcal{V}_{\mathfrak{h}} := \{ v \in L^2(\Omega) \,|\, \forall T \in \mathcal{T}_{\mathfrak{h}} \,,\; v_{|T} \in \mathbb{P}^d(T) \}$



Approximate solution : $V_{\mathfrak{h}}(\mathbf{x}, t) = \sum_{l=1}^{N_d} V_l(t)\theta_l(\mathbf{x}) \,, \quad \forall \mathbf{x} \in T \,.$

Local weak formulation (1)

$$\frac{\partial}{\partial t} V + \nabla . H(V, z) = S(V, z)$$

$\mathbb{P}^d(T) := \{$2 variables polynomials on T of degree at most $d\}$.

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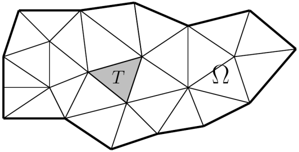

Approximate solution : $V_\mathfrak{h}(\mathbf{x}, t) = \sum_{l=1}^{N_d} V_l(t)\theta_l(\mathbf{x})\,, \quad \forall \mathbf{x} \in T\,.$

Local weak formulation (2)

$$\frac{\partial}{\partial t} V\phi_\mathfrak{h}(\mathbf{x}) + \nabla.H(V,z)\phi_\mathfrak{h}(\mathbf{x}) = S(V,z)\phi_\mathfrak{h}(\mathbf{x})$$

$\mathbb{P}^d(T) := \{$2 variables polynomials on T of degree at most $d\}$.

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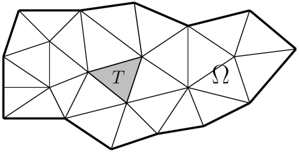


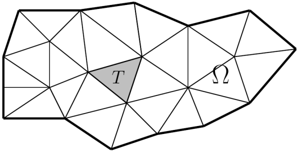
Approximate solution : $V_\mathfrak{h}(\mathbf{x}, t) = \sum_{l=1}^{N_d} V_l(t)\theta_l(\mathbf{x})\,, \quad \forall \mathbf{x} \in T\,.$

Local weak formulation (3)

$$\int_T \frac{\partial}{\partial t} V \phi_\mathfrak{h}(\mathbf{x}) d\mathbf{x} + \int_T \nabla.H(V,z)\phi_\mathfrak{h}(\mathbf{x}) d\mathbf{x} = \int_T S(V,z)\phi_\mathfrak{h}(\mathbf{x}) d\mathbf{x}$$

$\mathbb{P}^d(T) :=$ {2 variables polynomials on T of degree at most d} .

$\mathcal{V}_\mathfrak{h} := \{v \in L^2(\Omega) \,|\, \forall T \in \mathcal{T}_\mathfrak{h}\,,\ v_{|T} \in \mathbb{P}^d(T)\}$



Approximate solution : $V_\mathfrak{h}(\mathbf{x},t) = \sum_{l=1}^{N_d} V_l(t)\theta_l(\mathbf{x})\,, \quad \forall \mathbf{x} \in T\,.$

Local weak formulation (4)

$$\int_T \frac{\partial}{\partial t} V \phi_\mathfrak{h}(\mathbf{x}) d\mathbf{x} + \int_T \nabla.H(V,z)\phi_\mathfrak{h}(\mathbf{x}) d\mathbf{x} = \int_T S(V,z)\phi_\mathfrak{h}(\mathbf{x}) d\mathbf{x}$$

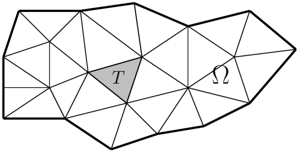
$$\int_T \frac{\partial}{\partial t} V \phi_\mathfrak{h}(\mathbf{x}) d\mathbf{x} - \int_T H(V,z).\nabla\phi_\mathfrak{h}(\mathbf{x}) d\mathbf{x} +$$

$$\int_{\partial T} H(V,z).\vec{n}\phi_\mathfrak{h}(s) ds = \int_T S(V,z)\phi_\mathfrak{h}(\mathbf{x}) d\mathbf{x}$$

dG method : general background

$\mathbb{P}^d(T) := \{$2 variables polynomials on T of degree at most $d\}$.

$\mathcal{V}_\mathfrak{h} := \{v \in L^2(\Omega) \,|\, \forall T \in \mathcal{T}_\mathfrak{h}\,,\; v_{|T} \in \mathbb{P}^d(T)\}$



Approximate solution : $V_\mathfrak{h}(\mathbf{x},t) = \sum_{l=1}^{N_d} V_l(t)\theta_l(\mathbf{x})\,, \quad \forall \mathbf{x} \in T\,.$

Through a semi-discrete formulation (1)

• $V \to V_\mathfrak{h}$

$$\int_T \frac{\partial}{\partial t}\Big(\sum_{l=1}^{N_d} V_l(t)\theta_l(\mathbf{x})\Big)\phi_\mathfrak{h}(\mathbf{x})d\mathbf{x} - \int_T H(V_\mathfrak{h}, z_\mathfrak{h}).\nabla\phi_\mathfrak{h}(\mathbf{x})d\mathbf{x} +$$

$$\int_{\partial T} H(V_\mathfrak{h}, z_\mathfrak{h}).\vec{n}\phi_\mathfrak{h}(s)ds = \int_T S(V_\mathfrak{h}, z_\mathfrak{h})\phi_\mathfrak{h}(\mathbf{x})d\mathbf{x}$$

$\mathbb{P}^d(T) := \{$2 variables polynomials on $T$of degree at most $d\}$.

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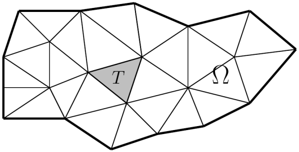

Approximate solution : $V_\mathfrak{h}(\mathbf{x},t) = \sum_{l=1}^{N_d} V_l(t)\theta_l(\mathbf{x})\,, \quad \forall \mathbf{x} \in T\,.$

Through a semi-discrete formulation (2)

- $V \to V_\mathfrak{h}$
- $\phi_\mathfrak{h} \to \theta_j$

$$\int_T \frac{\partial}{\partial t}\Big(\sum_{l=1}^{N_d} V_l(t)\theta_l(\mathbf{x})\Big)\theta_j(\mathbf{x})d\mathbf{x} - \int_T H(V_\mathfrak{h}, z_\mathfrak{h}).\nabla\theta_j(\mathbf{x})d\mathbf{x} +$$

$$\int_{\partial T} H(V_\mathfrak{h}, z_\mathfrak{h}).\vec{n}\theta_j(s)ds = \int_T S(V_\mathfrak{h}, z_\mathfrak{h})\theta_j(\mathbf{x})d\mathbf{x}$$

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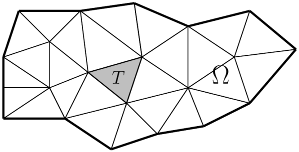

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- $V \to V_\mathfrak{h}$
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$$\int_T \frac{\partial}{\partial t}\left(\sum_{l=1}^{N_d} V_l(t)\theta_l(\mathbf{x})\right)\theta_j(\mathbf{x})d\mathbf{x} - \int_T H(V_\mathfrak{h}, z_\mathfrak{h}).\nabla\theta_j(\mathbf{x})d\mathbf{x} +$$

$$\sum_{k=1}^{3}\int_{\Gamma_{ij(k)}} H(V_\mathfrak{h}, z_\mathfrak{h}).\vec{n}_{ij(k)}\theta_j(s)ds = \int_T S(V_\mathfrak{h}, z_\mathfrak{h})\theta_j(\mathbf{x})d\mathbf{x}$$

Contributions on the edges

$$\int_{\Gamma_{ij(k)}} H(V_{\mathfrak{h}}, z_{\mathfrak{h}}).\vec{n}_{ij(k)}\theta_j(s)ds\,.$$

Numerical fluxes

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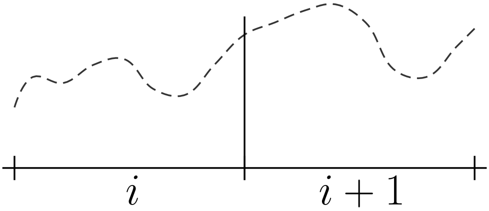

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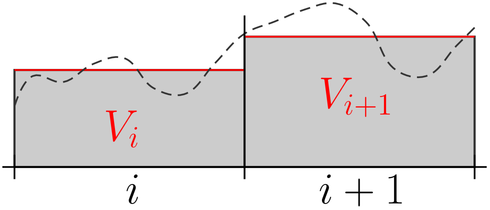

VF

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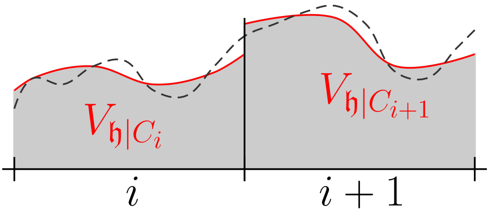

DG

Contributions on the edges

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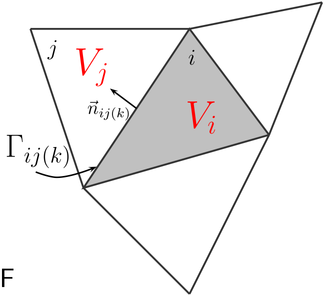

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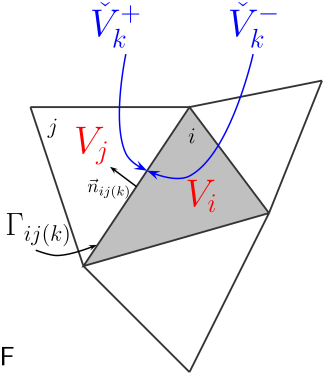

VF

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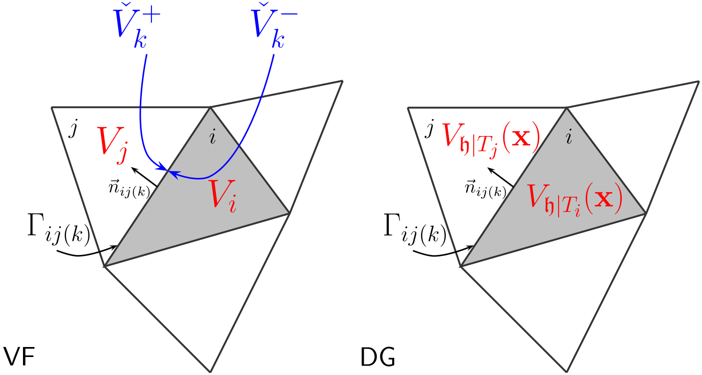

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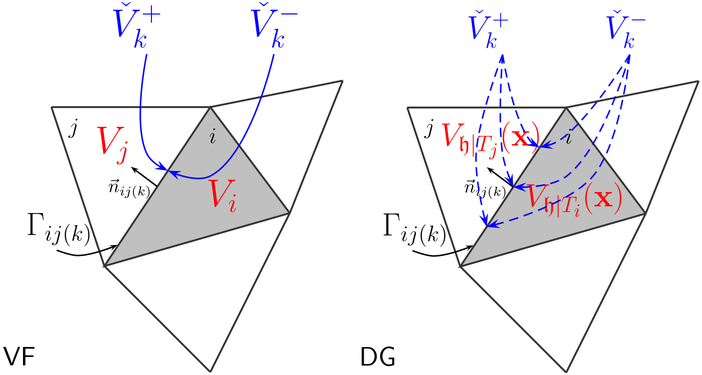

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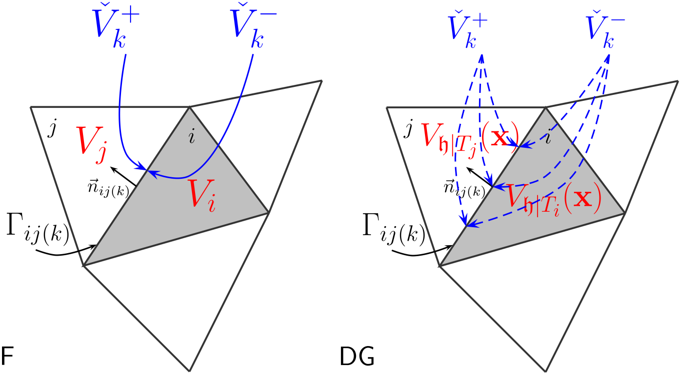


$\rightarrow$ Preservation of the motionless steady states.

Preservation of the water depth positivity

dG schemes and maximum principle

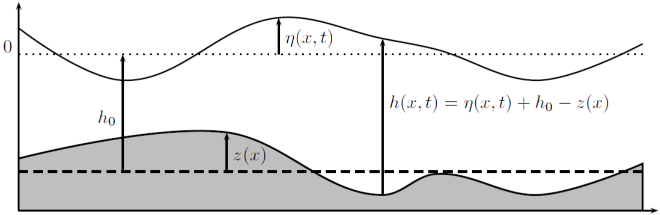
- [**X. Zhang and C.-W. Shu**. On maximum-principle-satisfying high order schemes for scalar conservation laws. J. Comput. Phys., 2010.]. dG method with maximum principle. 1D et 2D structured meshes, application to Euler equations.
- [**Y. Xing and X. Zhang and C.-W. Shu**. Positivity-preserving high order well-balanced discontinuous Galerkin methods for the shallow water equations. Adv. Wat. Res., 2010.]. Application to 1D SW equations.
- [**Y. Xing and X. Zhang**. Positivity-Preserving Well-Balanced Discontinuous Galerkin Methods for the Shallow Water Equations on unstructured triangular meshes. J. Sci. Comp., 2013.]. Extension to triangular meshes.

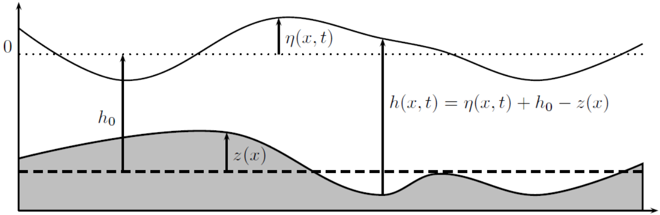
The method

Reduces to the study of a convex combination of first order Finite Volume schemes using appropriate quadrature points.

Outline

Model presentation





- [**D. Lannes, F. Marche**. A new class of fully nonlinear and weakly dispersive Green-Naghdi models for efficient 2D simulations. JCP, February 2015, 38–268.]

Revoke the time dependency

$$\begin{cases} \partial_t \eta + \partial_x(hu) = 0\,, \\ \left[1+\alpha\mathfrak{T}[h_b]\right]\left(\partial_t hu + \partial_x(hu^2) + \frac{\alpha-1}{\alpha} gh\partial_x\eta\right) + \frac{1}{\alpha} gh\partial_x\eta \\ \qquad + h\left(\mathcal{Q}_1(u) + g\mathcal{Q}_2(\eta)\right) + g\mathcal{Q}_3\left(\left[1+\alpha\mathfrak{T}[h_b]\right]^{-1}(gh\partial_x\eta)\right) = 0\,. \end{cases}$$

où $\mathfrak{T}[h]w = -\frac{h^3}{3}\partial_x{}^2\left(\frac{w}{h}\right) - h^2\partial_x h\partial_x\left(\frac{w}{h}\right)$, $h_b = h_0 - z$,

$\mathcal{Q}_{i=1,2,3}$: non linear, non conservative terms with second order derivatives.

Isolation of the hyperbolic part

A convenient formulation

$$\underbrace{\frac{\partial}{\partial t}U + \partial_x G(U) = B(U,z)}_{\text{Shallow Water}} + \underbrace{\mathfrak{D}(U,z)}_{\text{Dispersive terms}}$$

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1D Shallow Water equations :

$$U = \begin{pmatrix} h \\ hu \end{pmatrix} \quad , \quad G(U) = \begin{pmatrix} hu \\ gh^2 + hu^2 \end{pmatrix} \quad , \quad B(U) = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix} .$$

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Dispersive terms :

$$\mathfrak{D}(V,z) = \begin{pmatrix} 0 \\ \mathfrak{D}_{hu}(V,z) \end{pmatrix} , \text{ with}$$

$$\mathfrak{D}_{hu}(V,z) = \left[1 + \alpha\mathfrak{T}[h_b]\right]^{-1}\Big(\frac{1}{\alpha}gh\partial_x\eta + h\big(\mathcal{Q}_1(u) + g\mathcal{Q}_2(\eta)\big)$$
$$+ g\mathcal{Q}_3\big(\left[1 + \alpha\mathfrak{T}[h_b]\right]^{-1}(gh\partial_x\eta)\big)\Big) - \frac{1}{\alpha}gh\partial_x\eta .$$

Isolation of the hyperbolic part

A convenient formulation

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- Hyperbolic part : ok
- Dispersive part :

$$D_{\mathfrak{h}}(x,t) = \sum_{l=1}^{N_d} D_l(t)\theta_l(x)\,,\ x \in C_i\,.$$

- Well balancing and robustness ,
- Treatment of the second order derivatives .

A convenient formulation

$$\underbrace{\frac{\partial}{\partial t} U + \partial_x G(U) = B(U,z)}_{\text{Shallow Water}} + \underbrace{\color{red}{\mathfrak{D}(U,z)}}_{\text{Dispersive terms}}$$

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LDG formalism

Simplified case : $T = \partial_x{}^2$

Consider the second order ODE :

$$f - \partial_x^2 u = 0\,. \qquad (2)$$

(2) reduces to a coupled system of first order equations.

$$f + \partial_x v = 0 \quad , \quad v + \partial_x u = 0\,.$$

Weak formulation

$$\int_{x_i^l}^{x_i^r} f\phi_{\mathfrak{h}} - \int_{x_i^l}^{x_i^r} v\phi_{\mathfrak{h}}' + \widehat{v}_r\phi_{\mathfrak{h}}(x_i^r) - \widehat{v}_l\phi_{\mathfrak{h}}(x_i^l) = 0\,,$$

$$\int_{x_i^l}^{x_i^r} v\phi_{\mathfrak{h}} - \int_{x_i^l}^{x_i^r} u\phi_{\mathfrak{h}}' + \widehat{u}_r\phi_{\mathfrak{h}}(x_i^r) - \widehat{u}_l\phi_{\mathfrak{h}}(x_i^l) = 0\,.$$

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Weak formulation

$$\int_{x_i^l}^{x_i^r} f\phi_{\mathfrak{h}} - \int_{x_i^l}^{x_i^r} v\phi_{\mathfrak{h}}' + \widehat{v}_r\phi_{\mathfrak{h}}(x_i^r) - \widehat{v}_l\phi_{\mathfrak{h}}(x_i^l) = 0\,,$$

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LDG schemes :
[**B. Cockburn and C.-W. Shu**. The Local Discontinuous Galerkin method for time-dependent convection-diffusion systems. SIAM J. Numer. Anal., 1998].

Protocol : At each time step

- Detection : evaluation of $\mathbb{I}^k$ on each cell k.
- Determination of the breaking area.
- Switching strategy
  - Suppress of the dispersive terms on the targeted area.
  - Application of limiter to treat the hyperbolic part (Shallow Water).

Handling breaking waves

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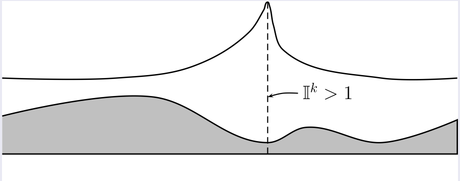

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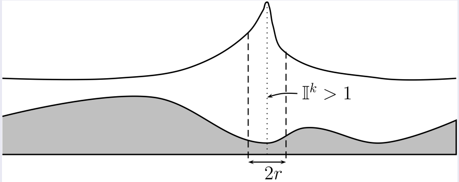

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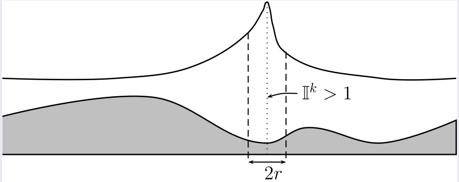

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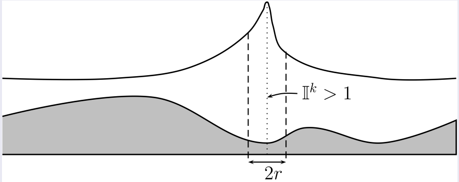

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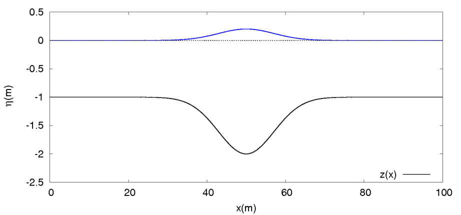
Convergence analysis and model comparison

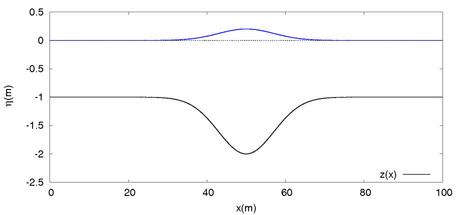
Profiles



Convergence rates

N	N_e 20	40	80	160	320	order
1	2.5e-1	4.2e-2	1.0e-3	2.8e-3	9.6e-4	1.9
2	7.5e-2	7.5e-3	6.2e-4	6.5e-5	7.7e-6	3.2
3	4.5e-3	3.0e-4	1.7e-5	9.4e-7	5.7e-8	4.0
4	7.0e-4	1.6e-5	4.6e-7	1.4e-8	4.4e-10	5.1
5	6.1e-5	7.6e-7	1.0e-8	1.6e-10	3.1e-12	6.1

Profiles



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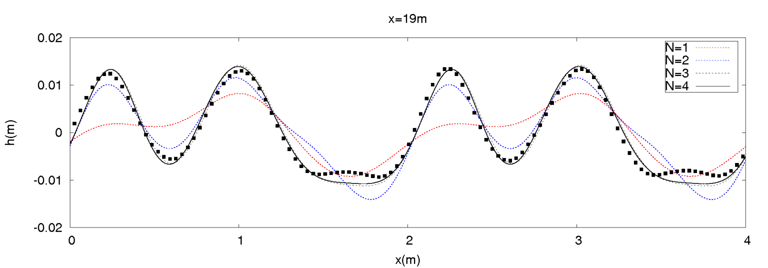
CPU time

Evolution of the ratio $\tau = \rho_o / \rho_c$.
ρ : mean iteration time (based on 1000 iterations).

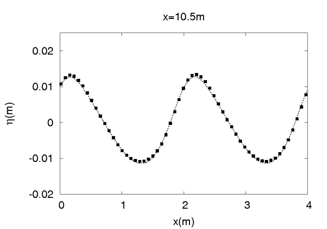
N	N_e 1000	2000	3000	4000	5000	6000
1	3.23	3.21	3.04	2.96	2.94	2.90
2	4.09	4.27	4.14	4.01	3.90	3.84
3	5.32	5.11	5.03	4.97	4.91	4.87
4	6.01	5.77	5.67	5.63	5.51	5.55
5	6.66	6.38	6.32	6.30	6.26	6.16
6	7.15	6.99	7.05	6.97	6.86	6.54

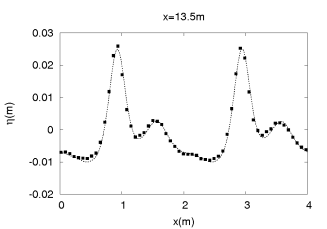
Propagation of highly dispersive waves

Effects of an increase of N (gauge 19)



gauges x=10.5, 13.5, 15.7, 17.3





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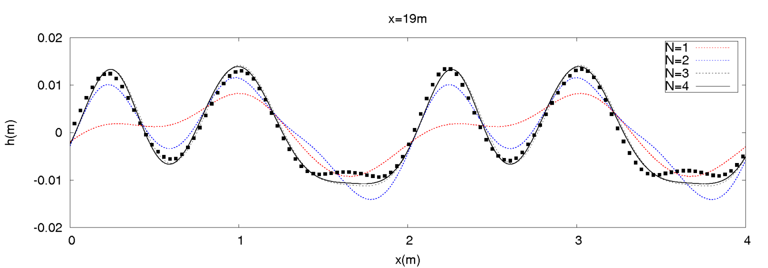

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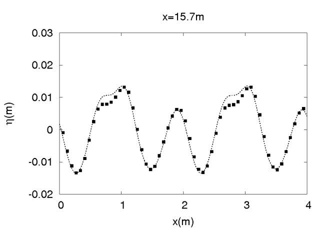

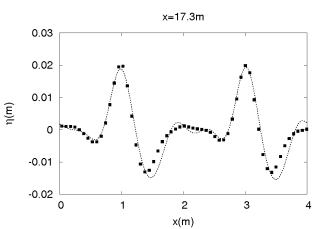

Outline

Perspectives

- Boundary conditions.
- Handling breaking waves, coupling GN/SW.
- 2D extension of the dispersive scheme on triangular meshes.

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Thank you !