Impact de gouttes: étalement, splashes, etc…

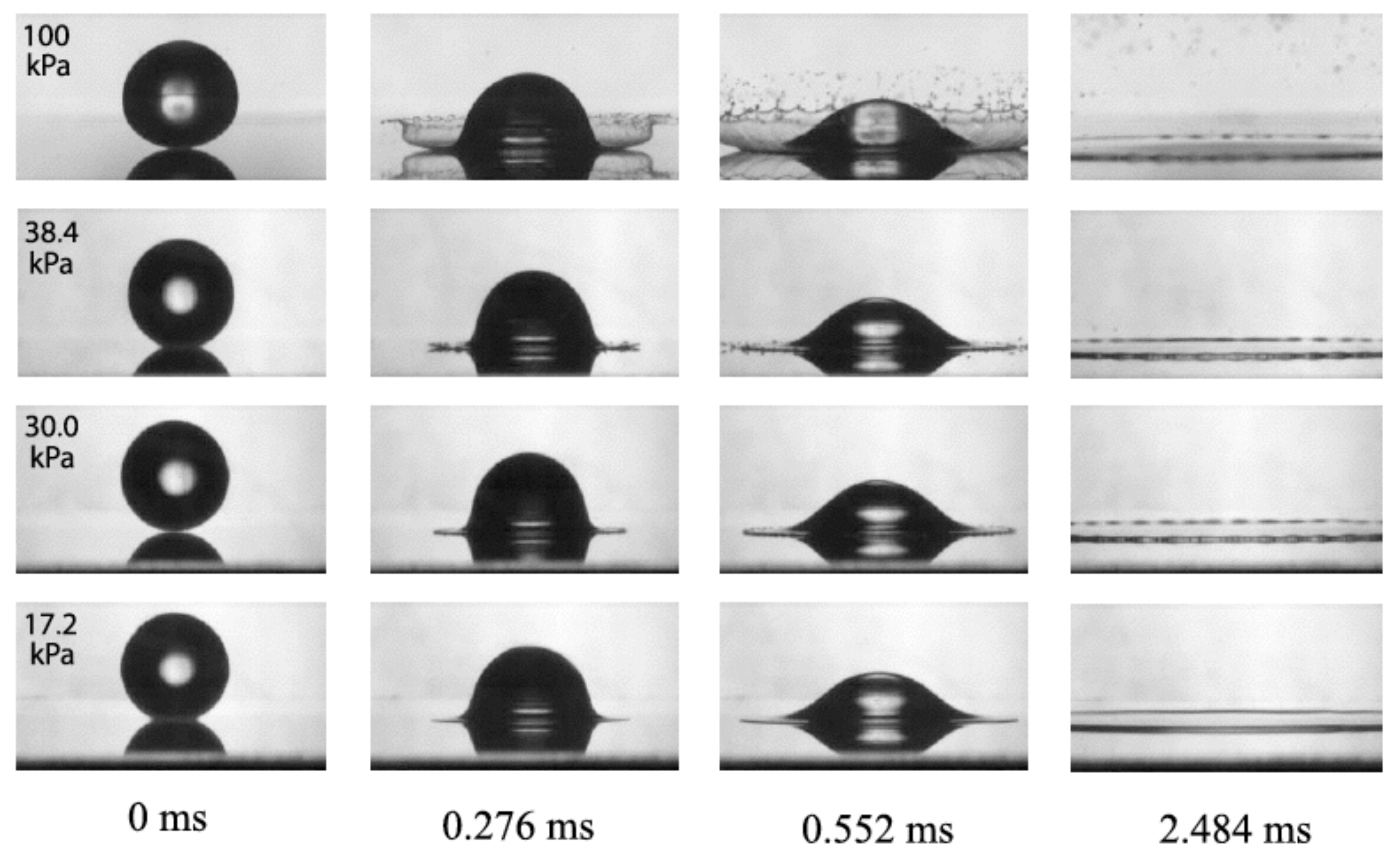
Christophe Josserand
Institut D'Alembert, CNRS-UPMC

General motivations

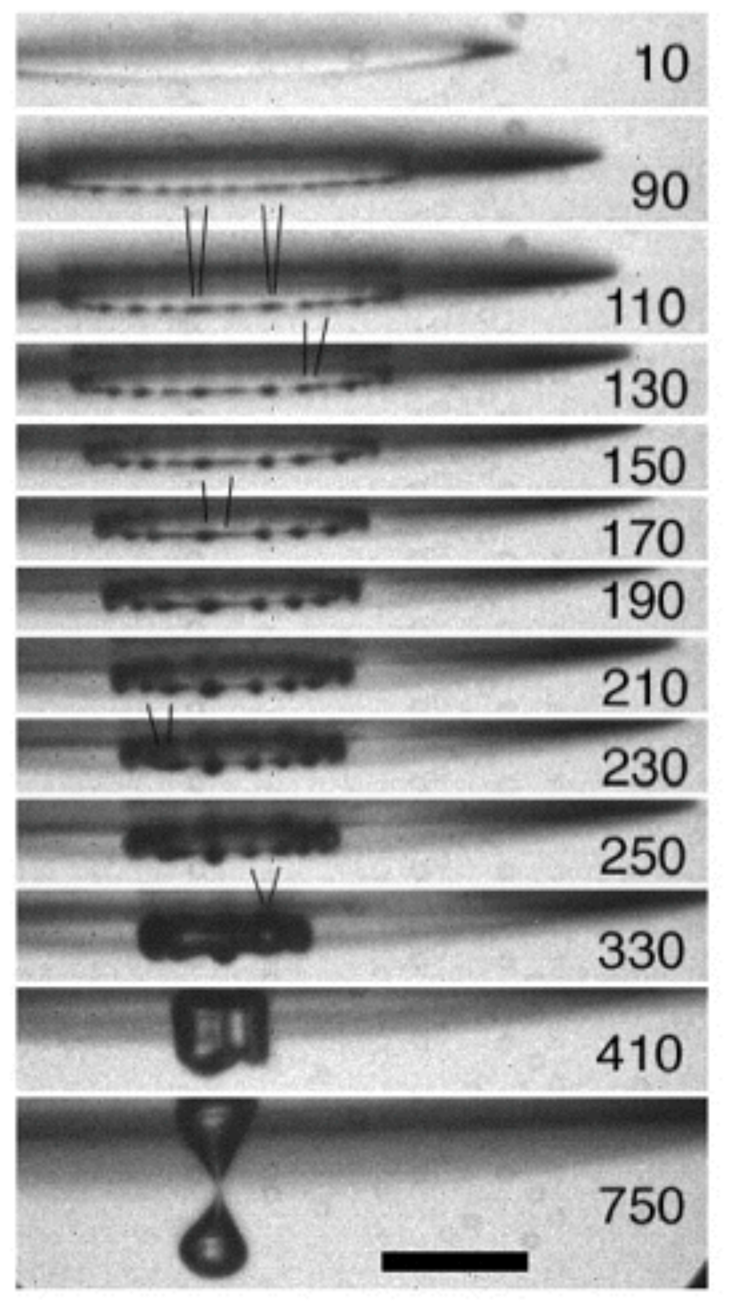
- Why drop impacts?
- Numerous contexts from everyday-life situations to industrial applications
- large spatial range: from ink-jet printers and nanojet to comets
- typical applications: raindrop, atomisation, combustion chambers...

Questions

- what controls the splashing
- what is the influence of the surrounding gas during impacts?
- important in the «late» time dynamics, corolla, droplets, etc...
- recently it has been shown to be crucial for the short time dynamics although it was usually neglected before.

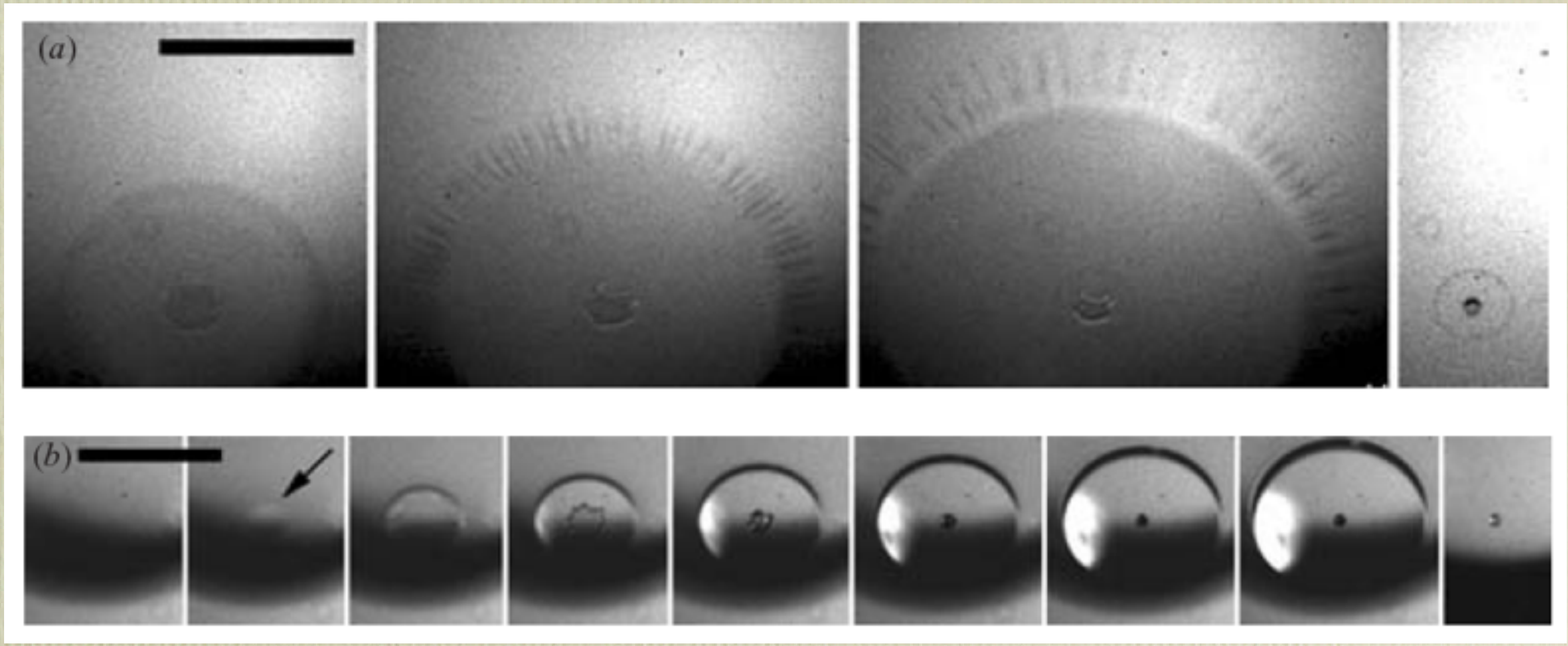


L. Xu , W.W. Zhang and S.R. Nagel, «Drop splashing on a dry smooth surface»,
Phys. Rev. Lett. 94, 184505 (2005).



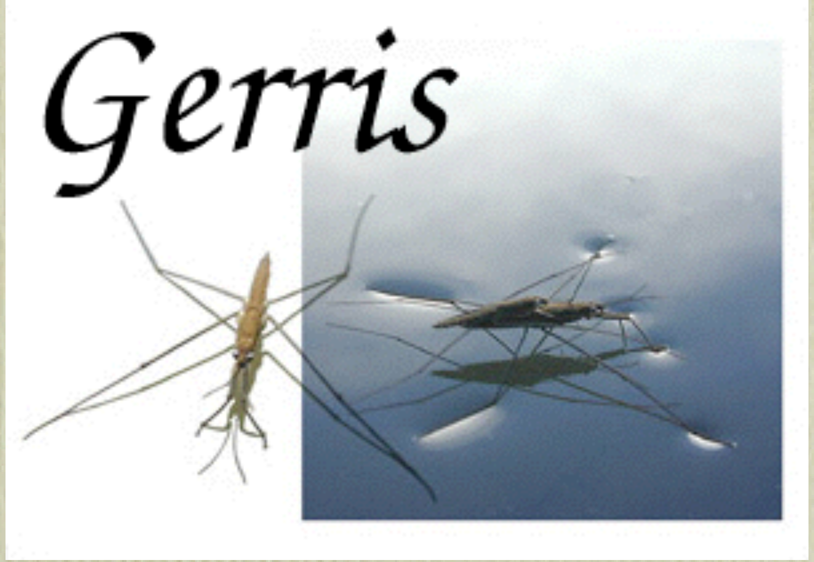
Thoroddsen, S. T., Etoh, T. G. & Takehara, K., 2003, «Air entrapment under and impacting drop.» J. Fluid Mech. Vol. 478, 125-134.

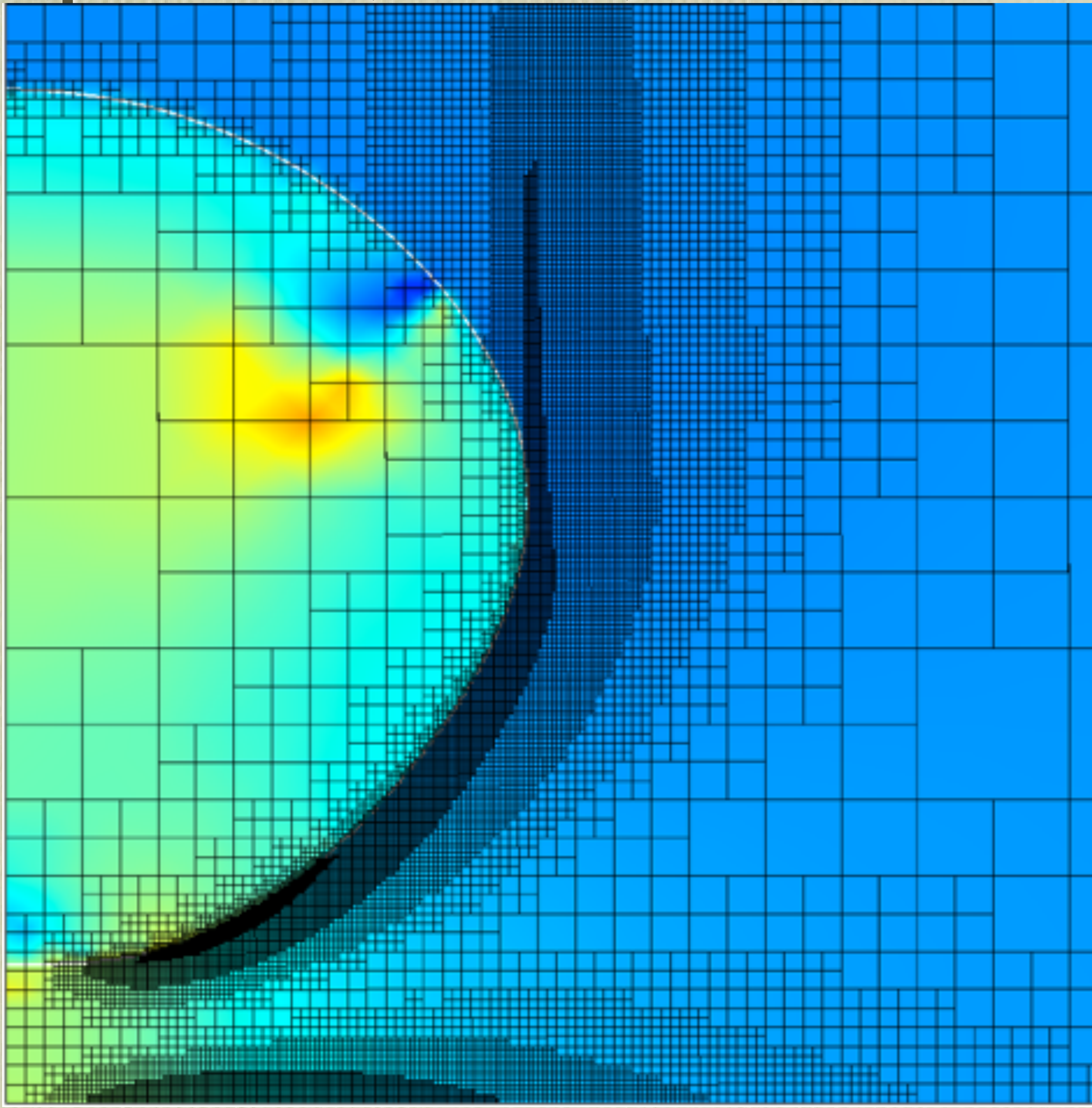
Also on solid surfaces.



S.T. Thoroddsen, T.G. Etoh, K. Takehara, N. Ootsuka and Y. Hatsuki, "The air bubble entrapped under a drop impacting on a solid surface", J. Fluid Mech. 545, 203-212 (2005).

Numerical simulations of drop impacts (DNS)

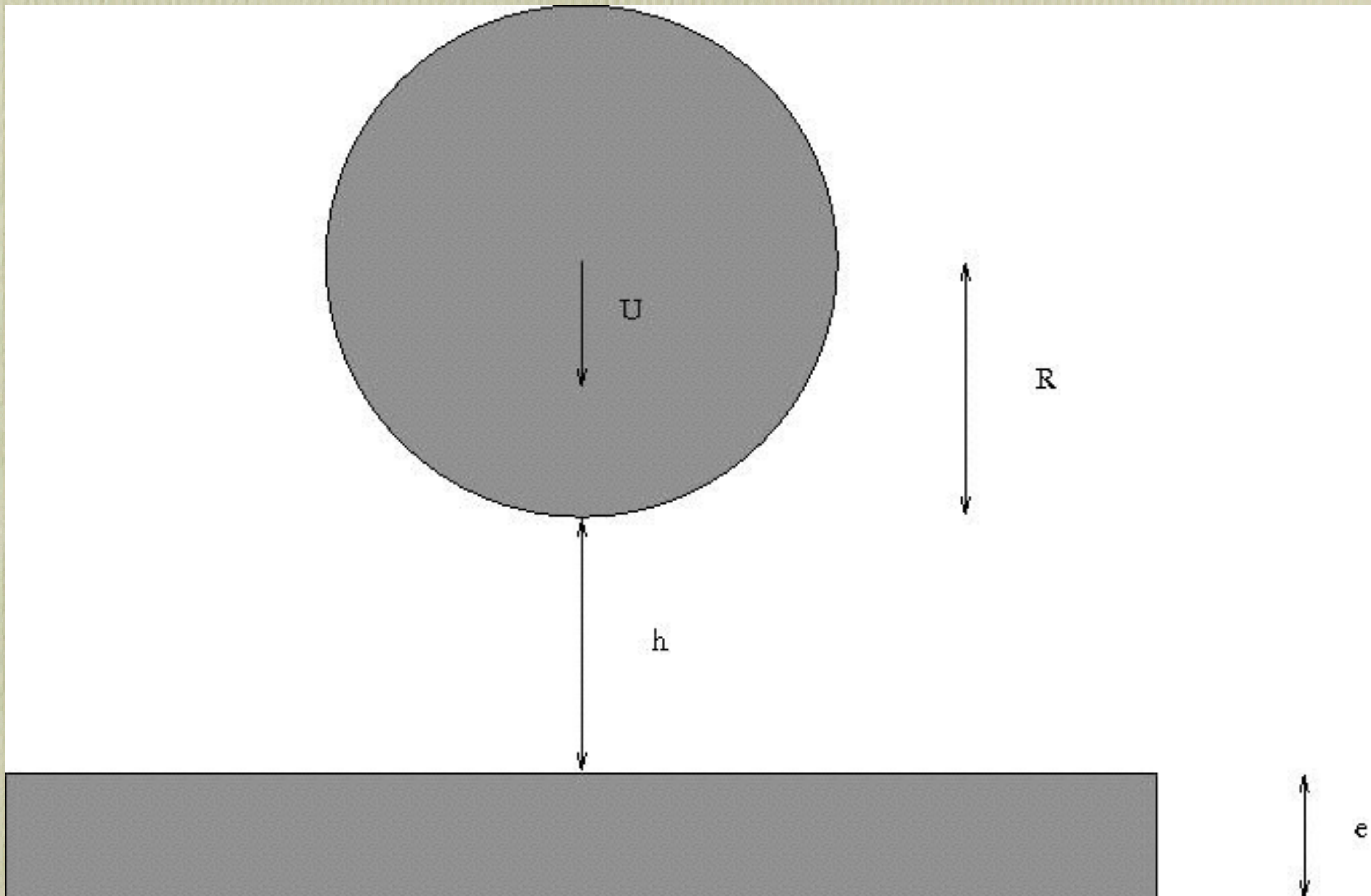




GERRIS hints

- 2 fluids incompressible Navier-Stokes equation with solid boundaries
- VOF method (PLIC interface reconstruction)
- Adaptive mesh refinement (quad-oct-tress, dynamical)
- multigrid Poisson solver

$$\rho(\frac{\partial u}{\partial t} + u \cdot \nabla u) = -\nabla p + \mu \Delta u + \sigma \kappa \delta_s n$$


U
R
h
e

Dimensionless numbers

$$Re_l = \frac{\rho_l U D}{\mu_l}$$

$$We = \frac{\rho_l U^2 D}{\gamma}$$

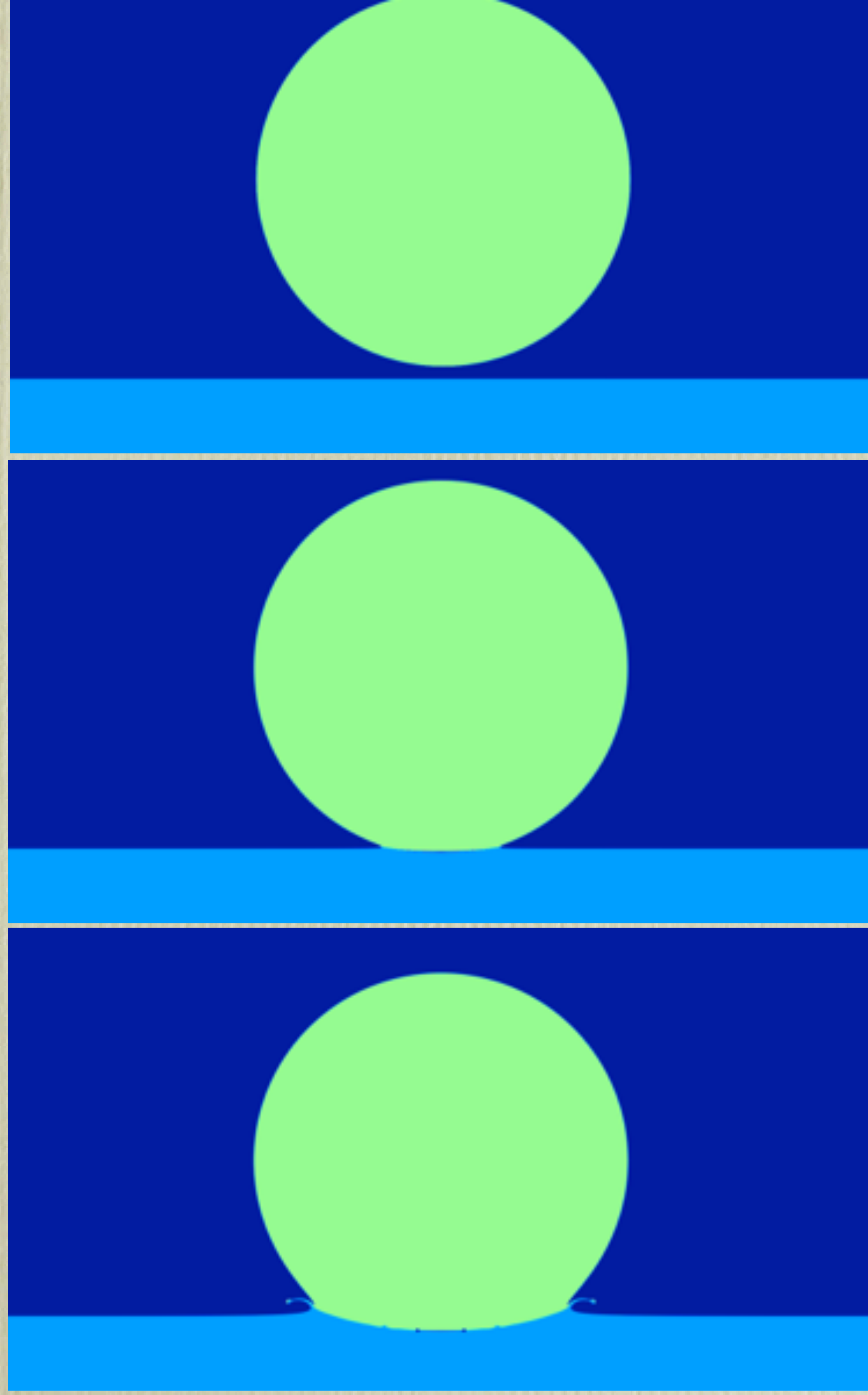
$$m = \frac{\mu_g}{\mu_l}$$

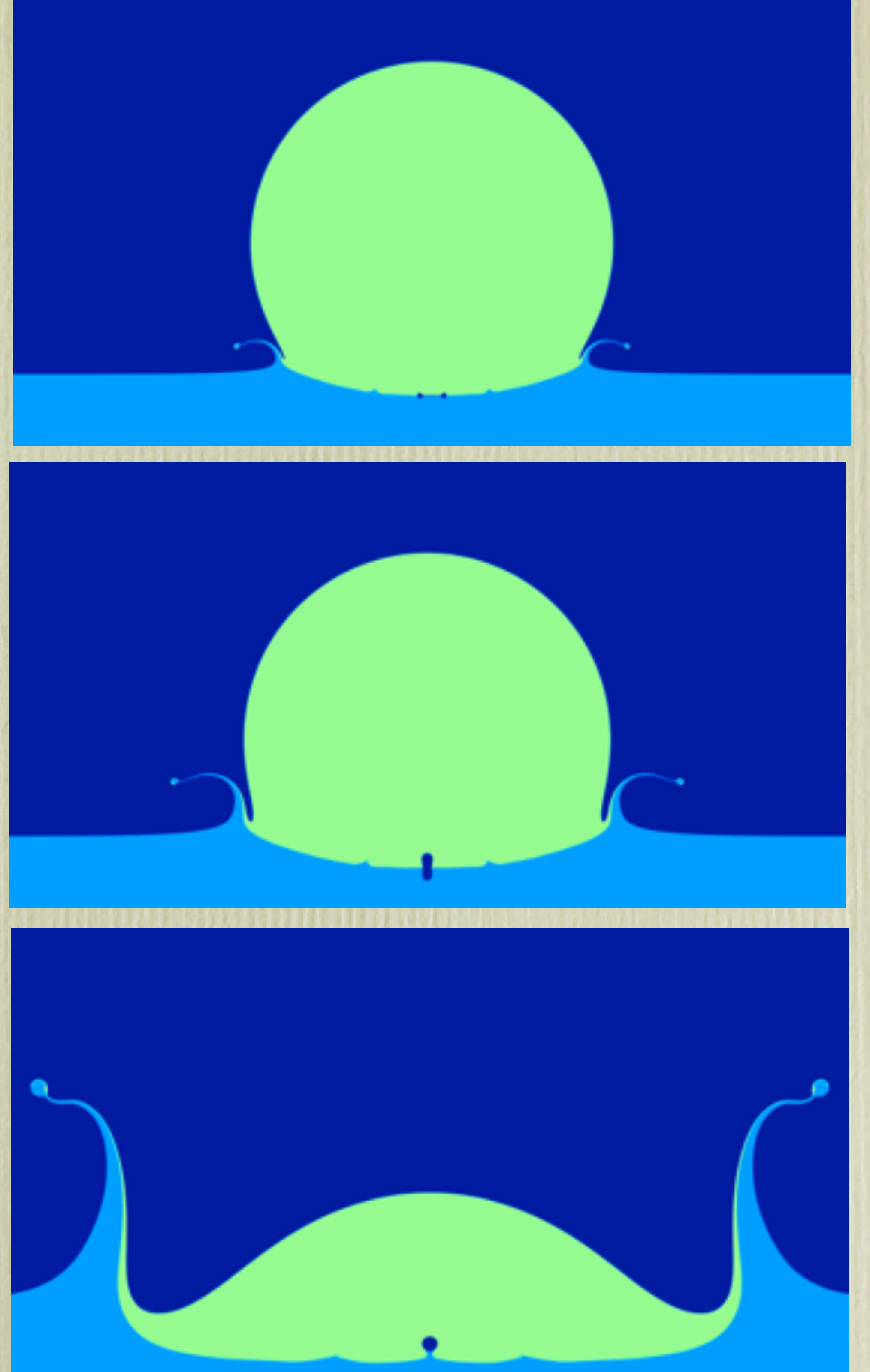
$$r = \frac{\rho_g}{\rho_l}$$

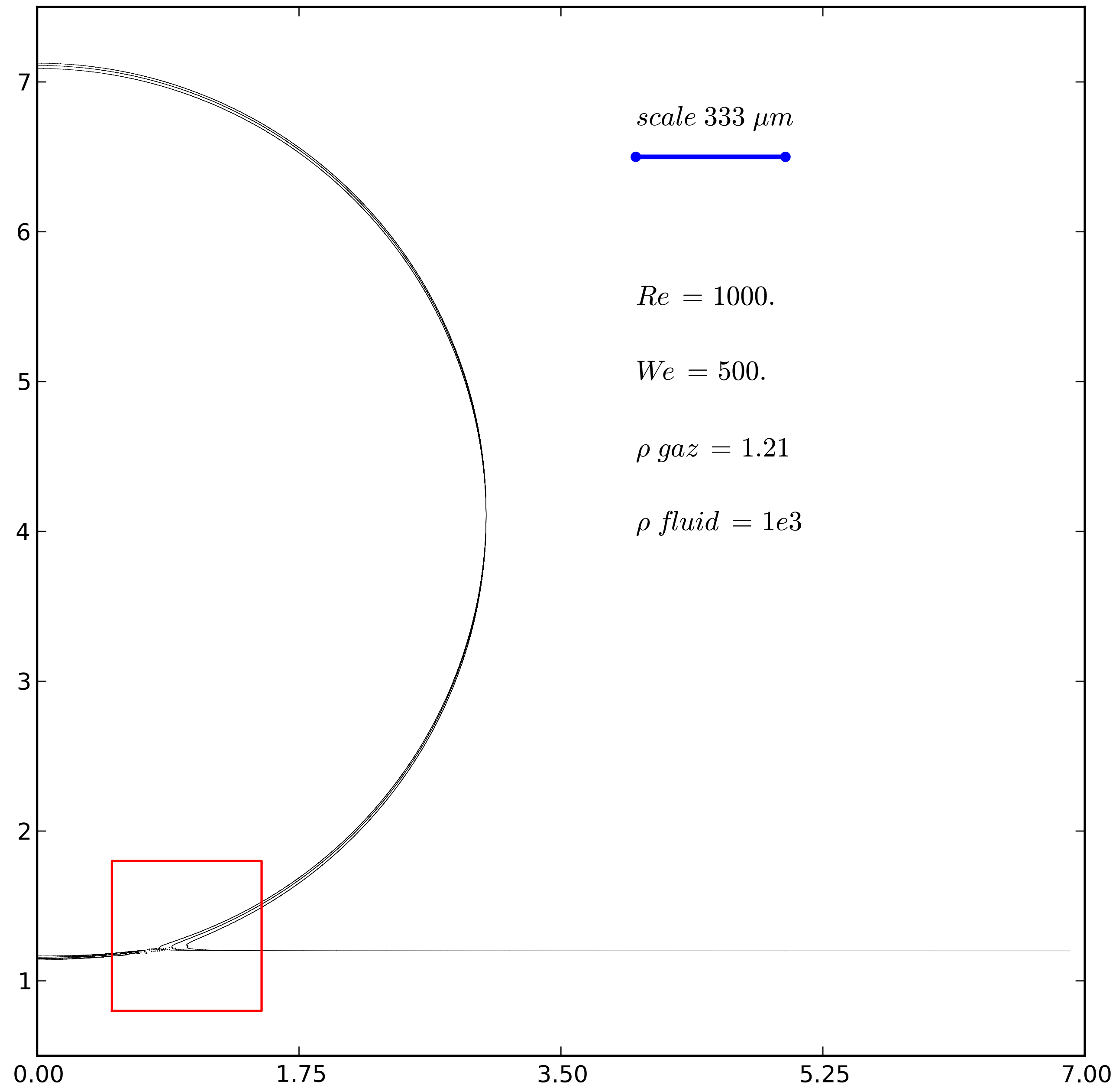
$$A = \frac{e}{D}$$

$$Re_g = \frac{\rho_g U D}{\mu_g}$$

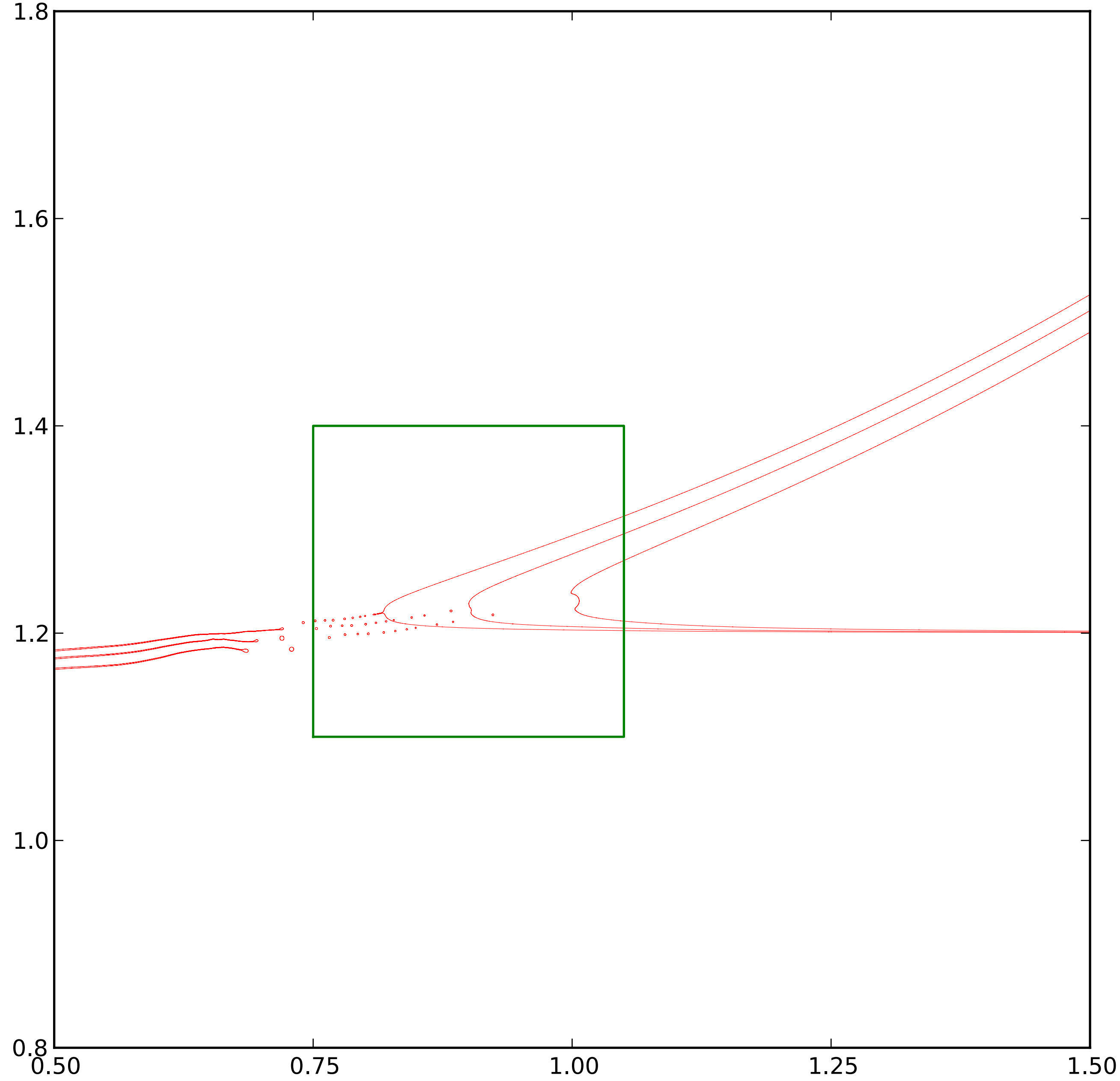
$$St = \frac{\mu_g}{\rho_l U D} = \frac{m}{Re_l}$$

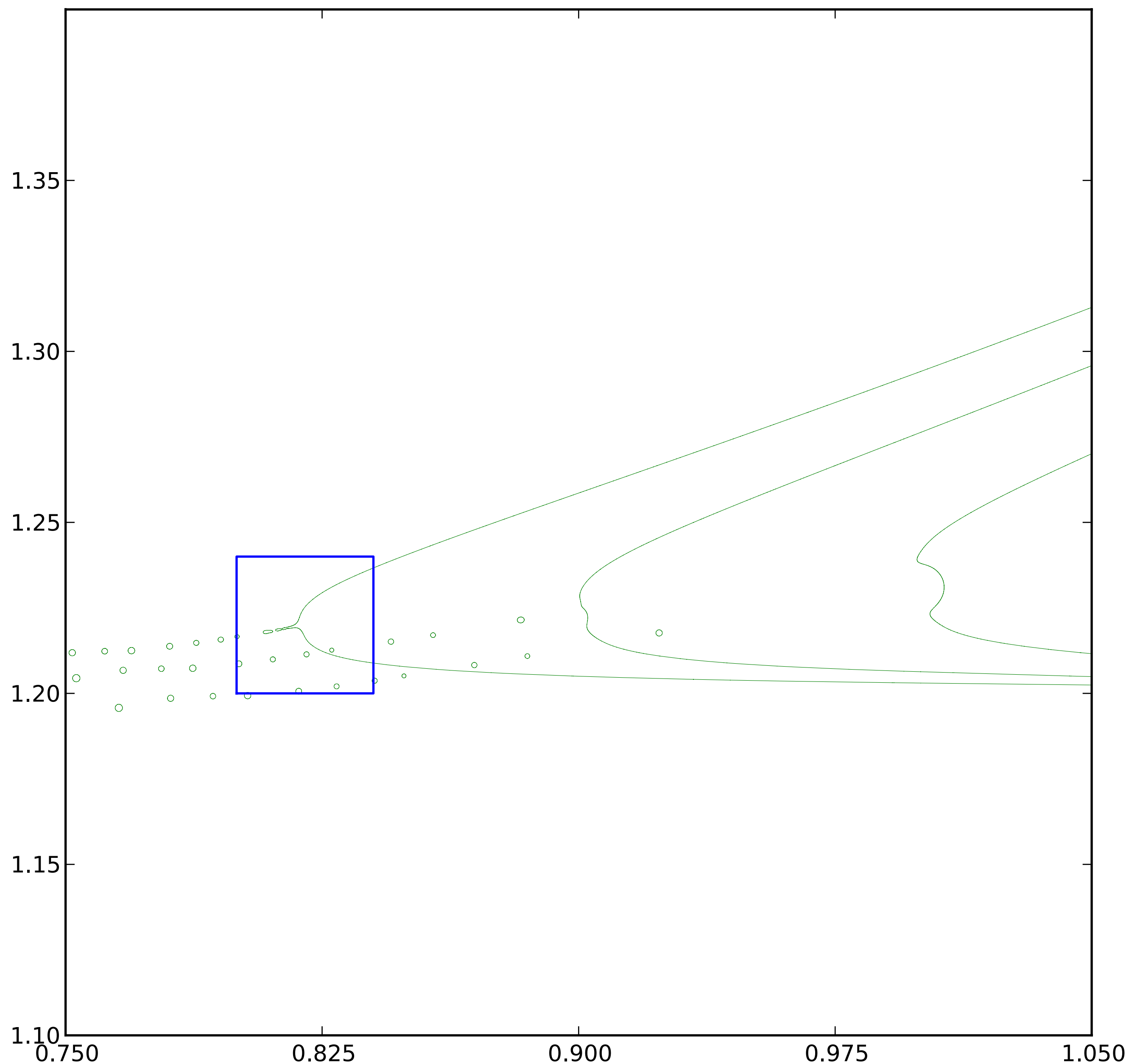


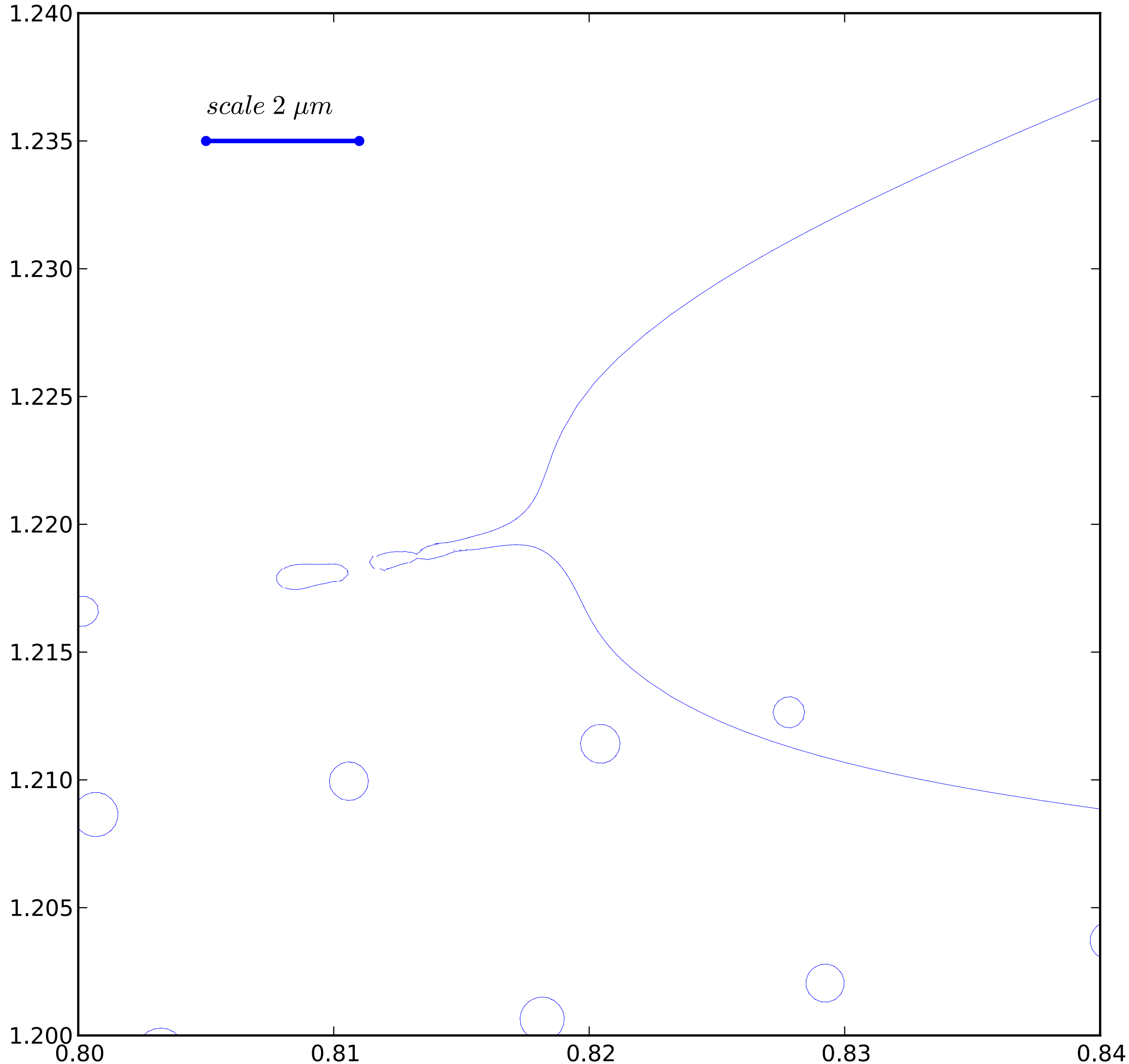




scale 333 μm
Re = 1000.
We = 500.
ρ gaz = 1.21
ρ fluid = 1e3
7
6
5
4
3
2
1
0.00
1.75
3.50
5.25
7.00


1.8
1.6
1.4
1.2
1.0
0.8
0.50
0.75
1.00
1.25
1.50

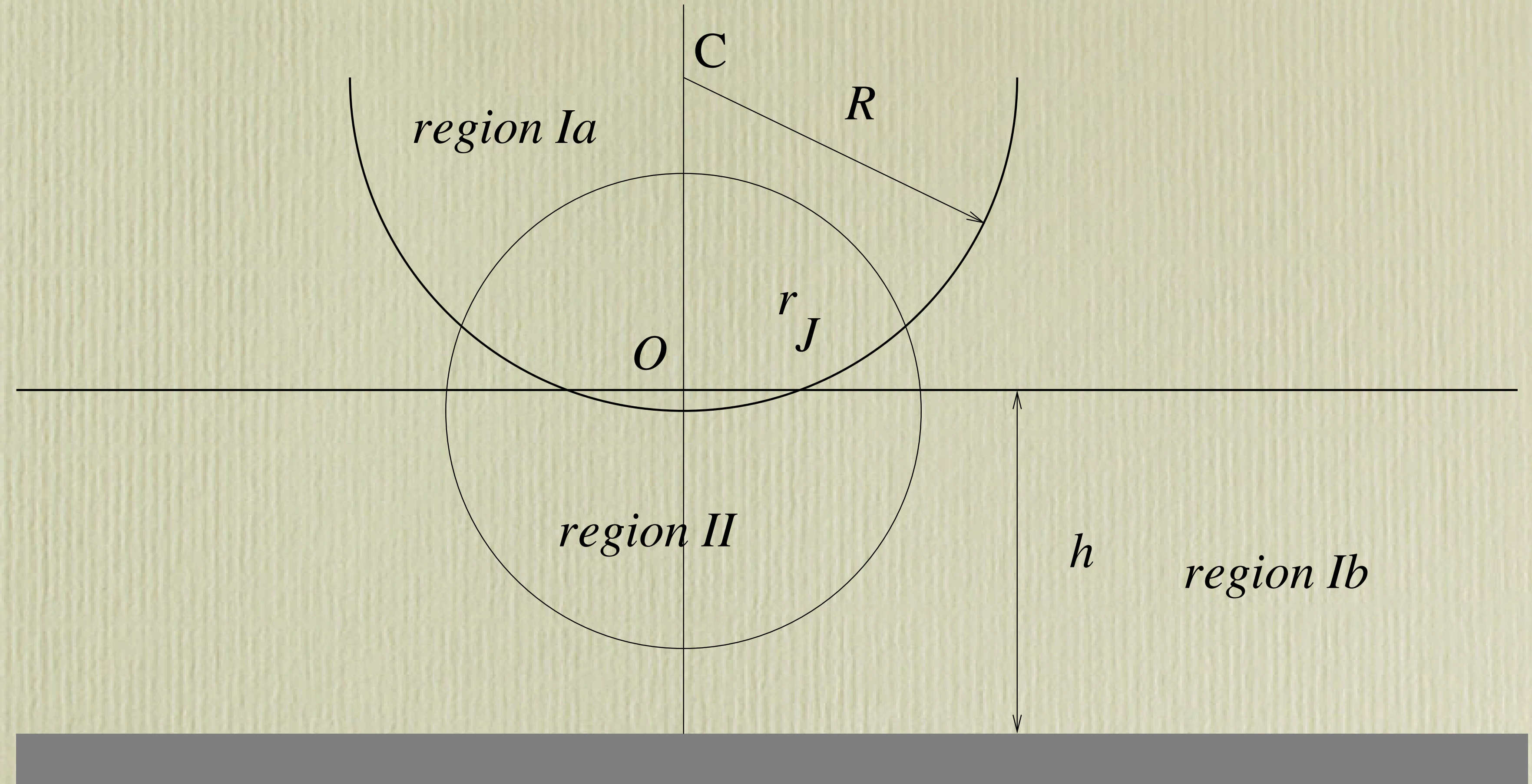

1.35
1.30
1.25
1.20
1.15
1.10
0.750
0.825
0.900
0.975
1.050

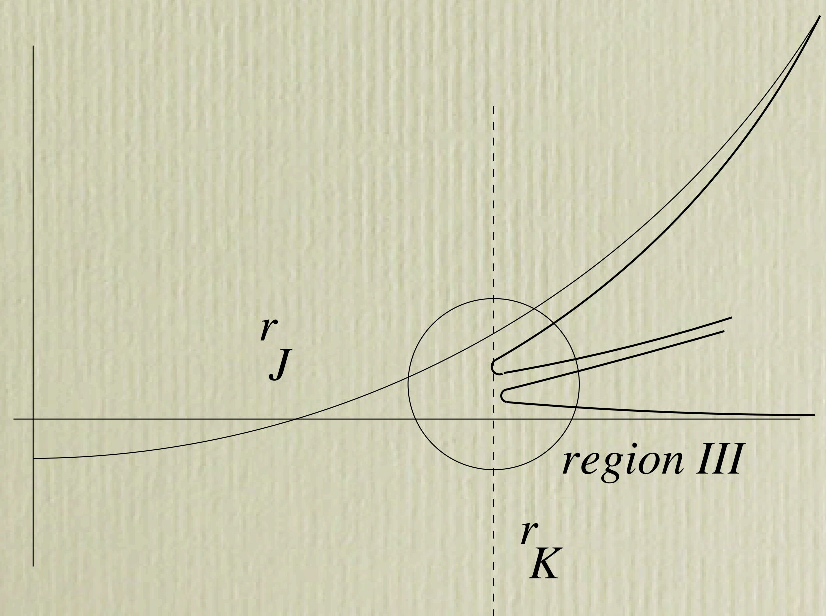


scale 2 μm
1.240
1.235
1.230
1.225
1.220
1.215
1.210
1.205
1.200
0.80
0.81
0.82
0.83
0.84

Scaling arguments

- as the drop impacts on the thin liquid sheets it decelerates:
- first because of the air cushioning
- then because of the liquid film
- finally because the liquid film is thin
- self-similar analysis at short time


C
R
region Ia
r J
O
region II
h
region Ib


r J
region III
r K

At the bottom of the drop $r \sim \sqrt{Dz}$

So that the impact radius follows $r_i \sim \sqrt{DUt}$

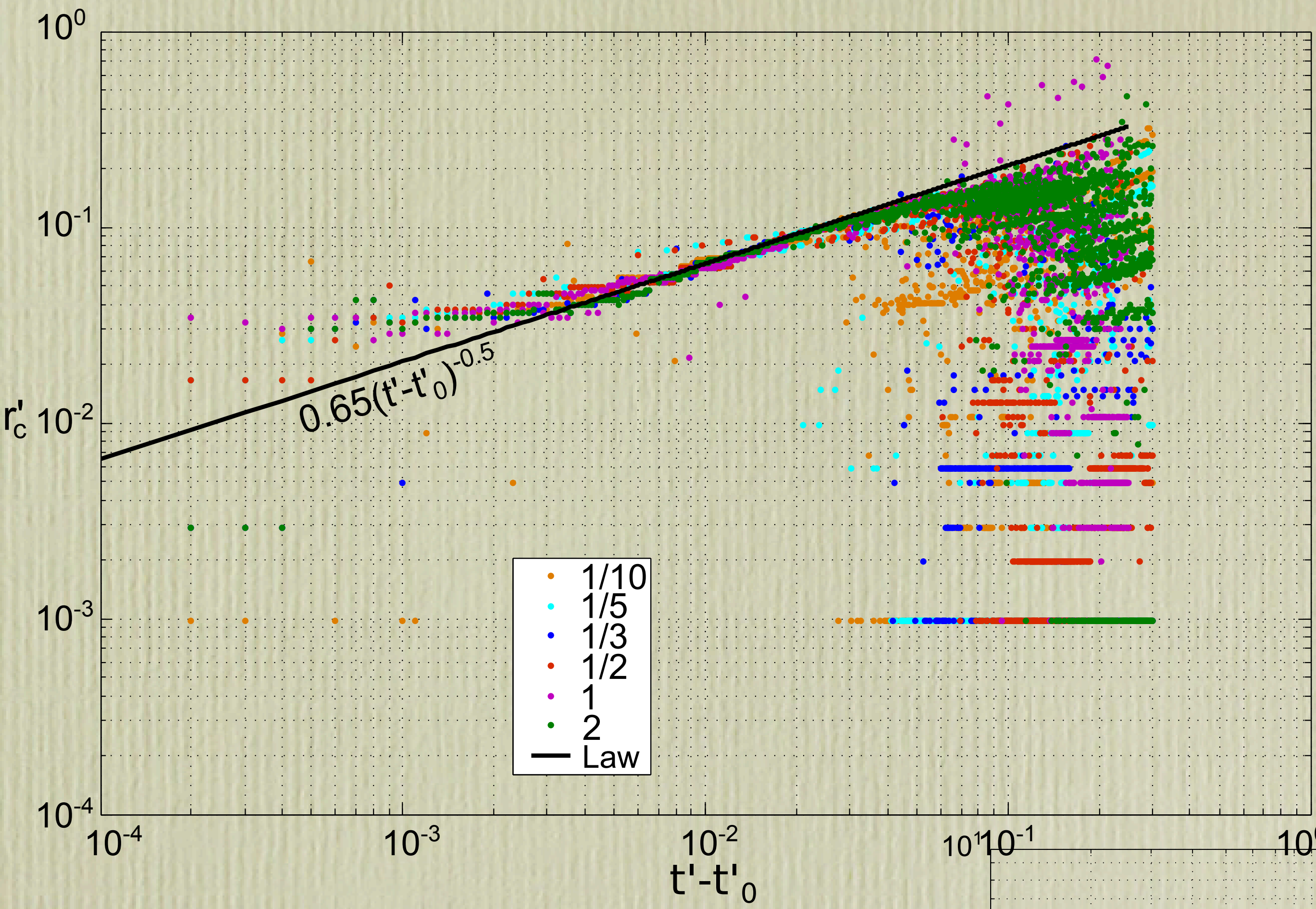
Mass flux inside the impact region

$$\delta\dot{m} \sim \frac{\delta(2\rho_l \pi r_i^3)}{\delta t} \sim 2\pi\rho_l r_i^2 \dot{r}_i$$

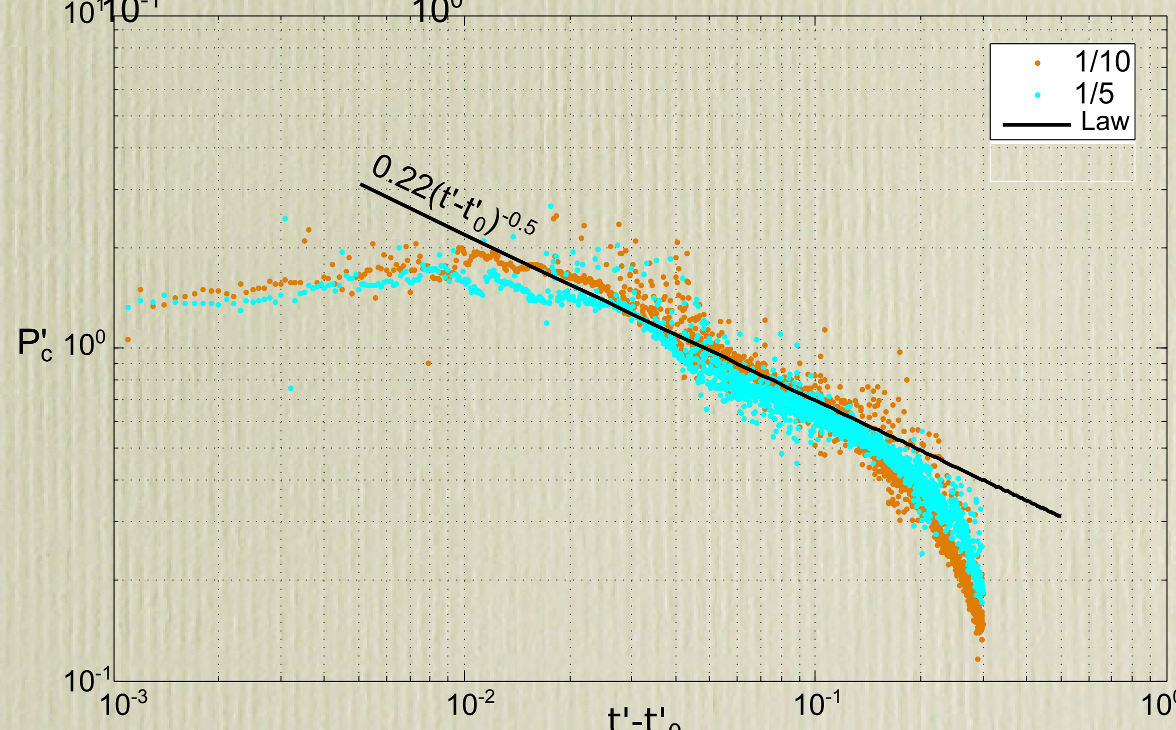
If this mass is transfered into a viscous jet

$$\delta\dot{m} \sim 2\pi\rho_l r_i e_j U_j \sim 2\pi\rho_l r_i \sqrt{\nu_l t}\, U_j$$

$$U_j \propto \sqrt{Re_l}\, U$$



10^0
10^{-1}
r'_c 10^{-2}
10^{-3}
10^{-4}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
$t'-t'_0$
$0.65(t'-t'_0)^{-0.5}$
1/10
1/5
1/3
1/2
1
2
Law



P'_c 10^0
10^{-1}
10^{-3}
10^{-2}
10^{-1}
10^0
$0.22(t'-t'_0)^{-0.5}$
1/10
1/5
Law

Validity of the scaling theory?

- condition for the jet to emerge:
- has to win against capillary effect
- has to go faster than the geometrical intersection
- the air cushioning should perturb (delay) the whole dynamics

Jet formation

$$U_j \gg v_{TC}$$

$$v_{TC} = \sqrt{\frac{\gamma}{\rho_l e_j}}$$

$$\frac{Ut}{D} \gg \frac{1}{Re_l \cdot We^2}$$

$$U_j \gg \dot{r}_i$$

$$\frac{Ut}{D} \gg \frac{1}{Re_l}$$

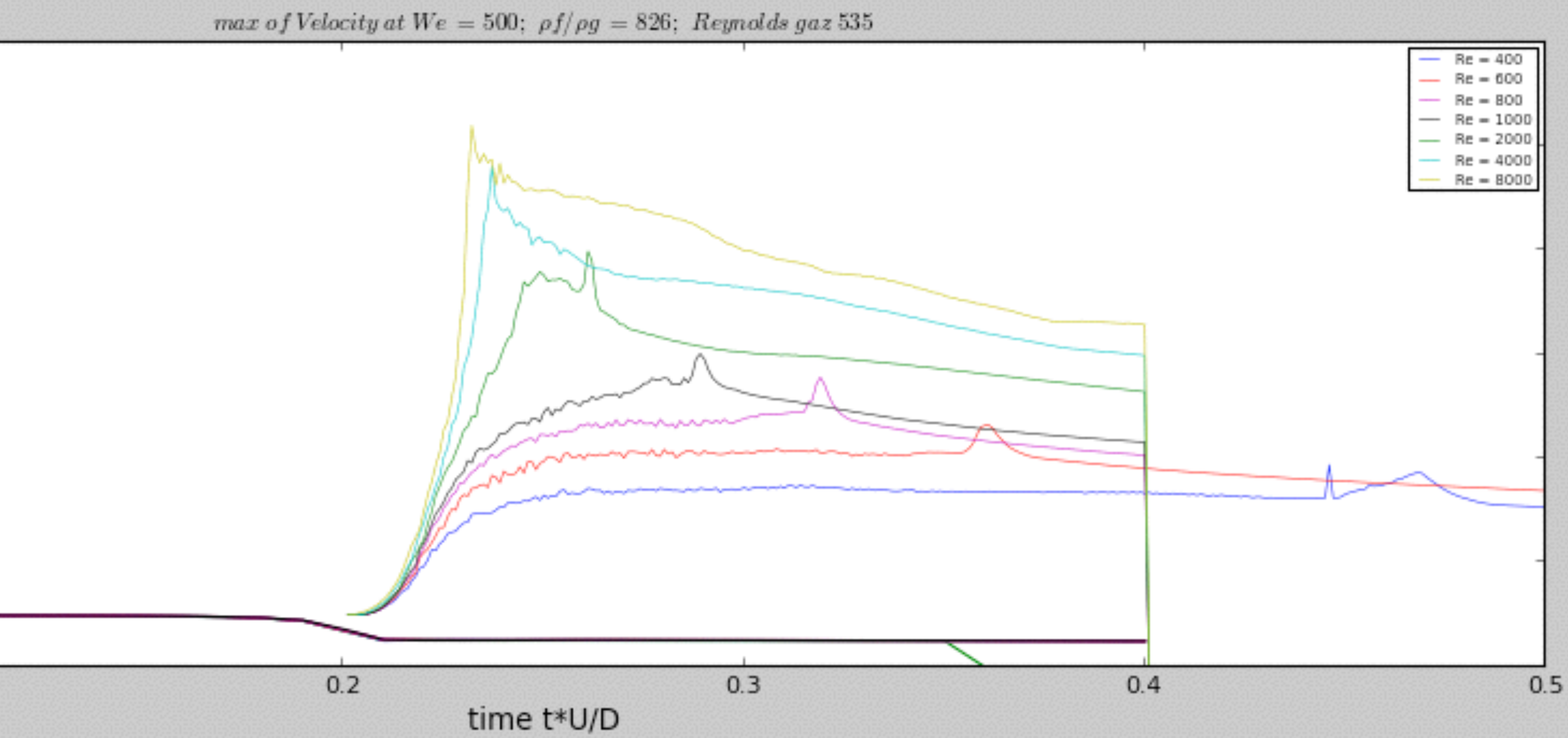
Air cushioning

Lubrication regime $$\frac{\partial h}{\partial t} = \frac{1}{12\mu r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial p}{\partial r}\right)$$

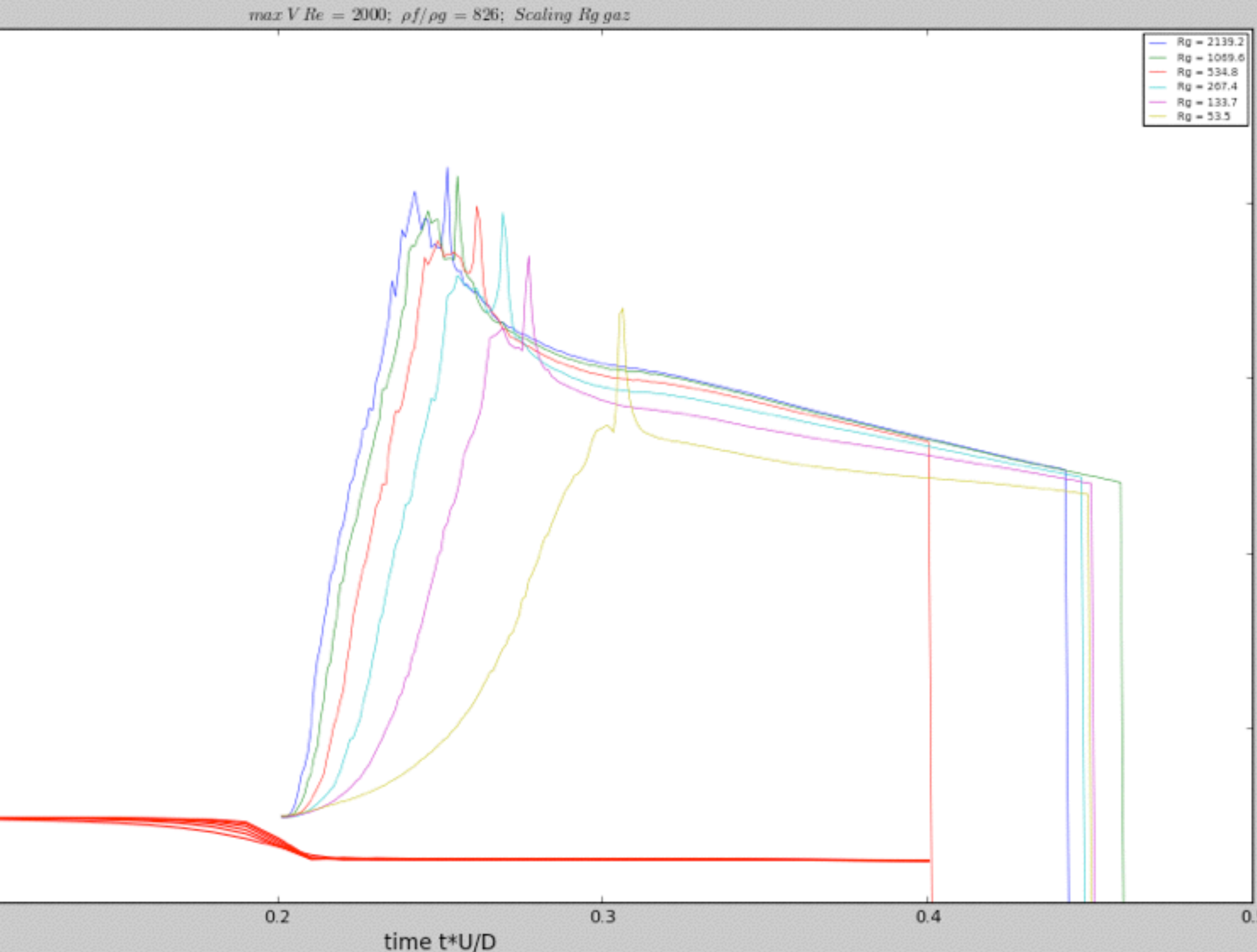
$$P_{lub} \sim \frac{\mu D}{U t^2}$$

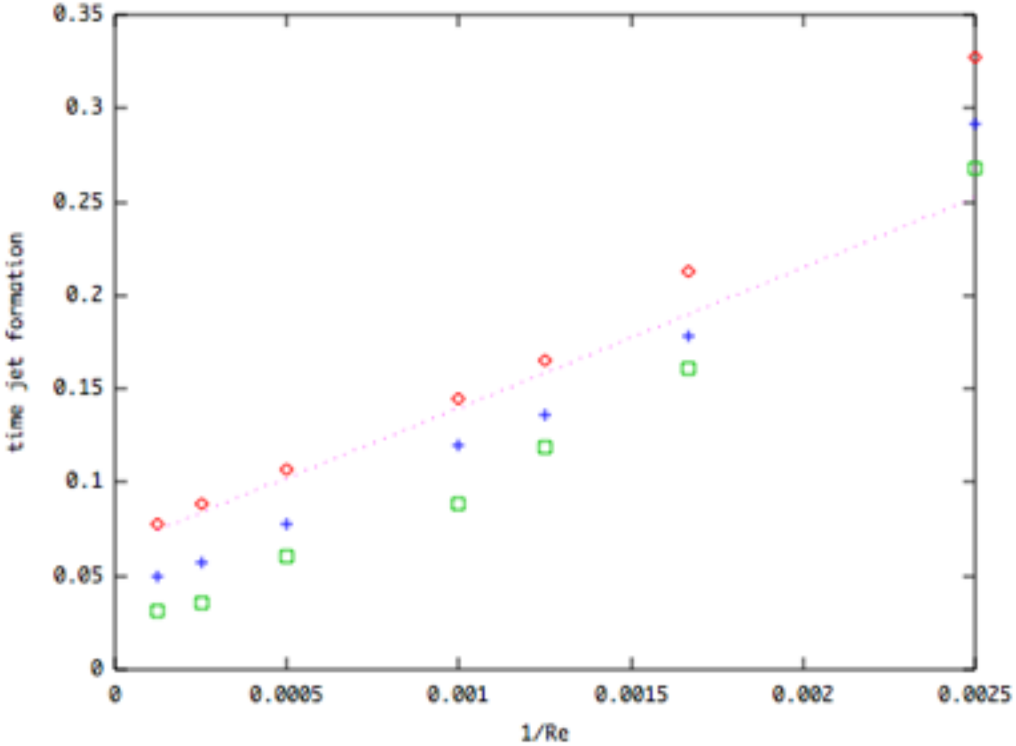
$$P_{iner} \sim \frac{\delta \dot{m} U}{\pi r_i^2} \sim \rho_l U \dot{r}_i$$

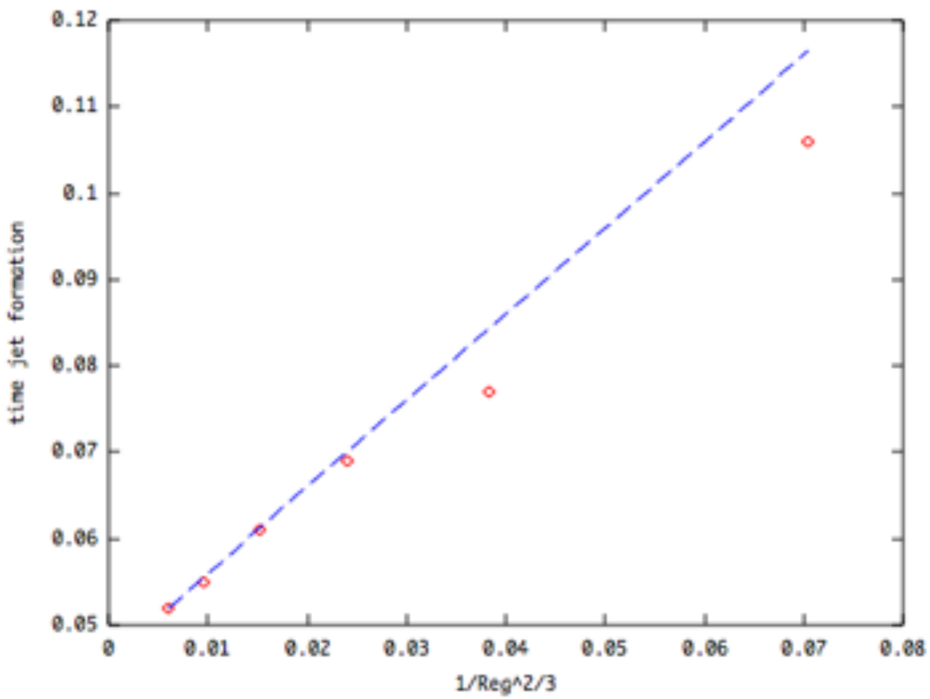
$$P_{iner} \sim P_{lub} \quad \rightarrow \quad \frac{Ut}{D} \propto St^{2/3}$$

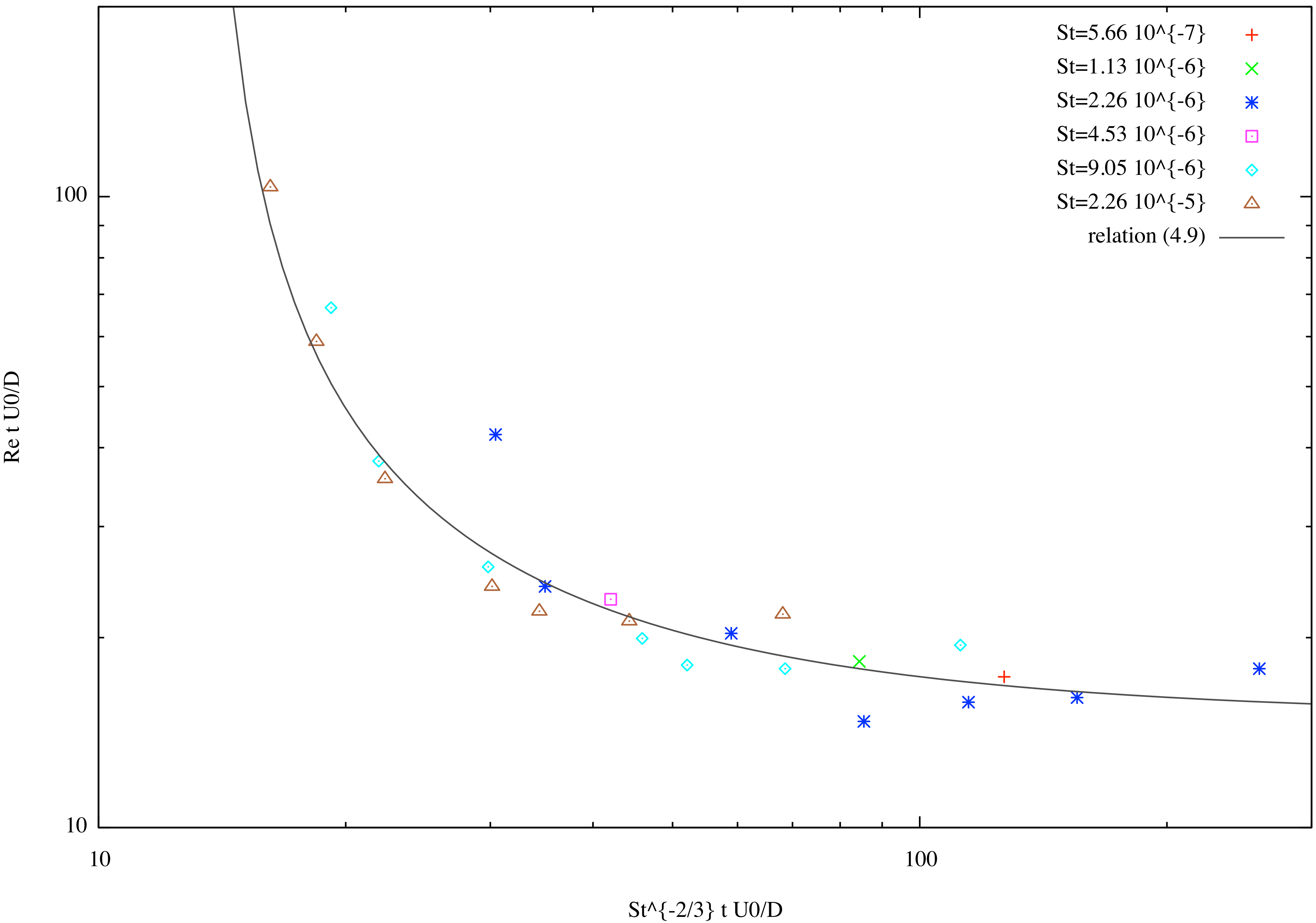


$max\ of\ Velocity\ at\ We\ =500;\ \rho f/\rho g\ =826;\ Reynolds\ gaz\ 535$
Re = 400
Re = 600
Re = 800
Re = 1000
Re = 2000
Re = 4000
Re = 8000
0.2
0.3
0.4
0.5
time t*U/D

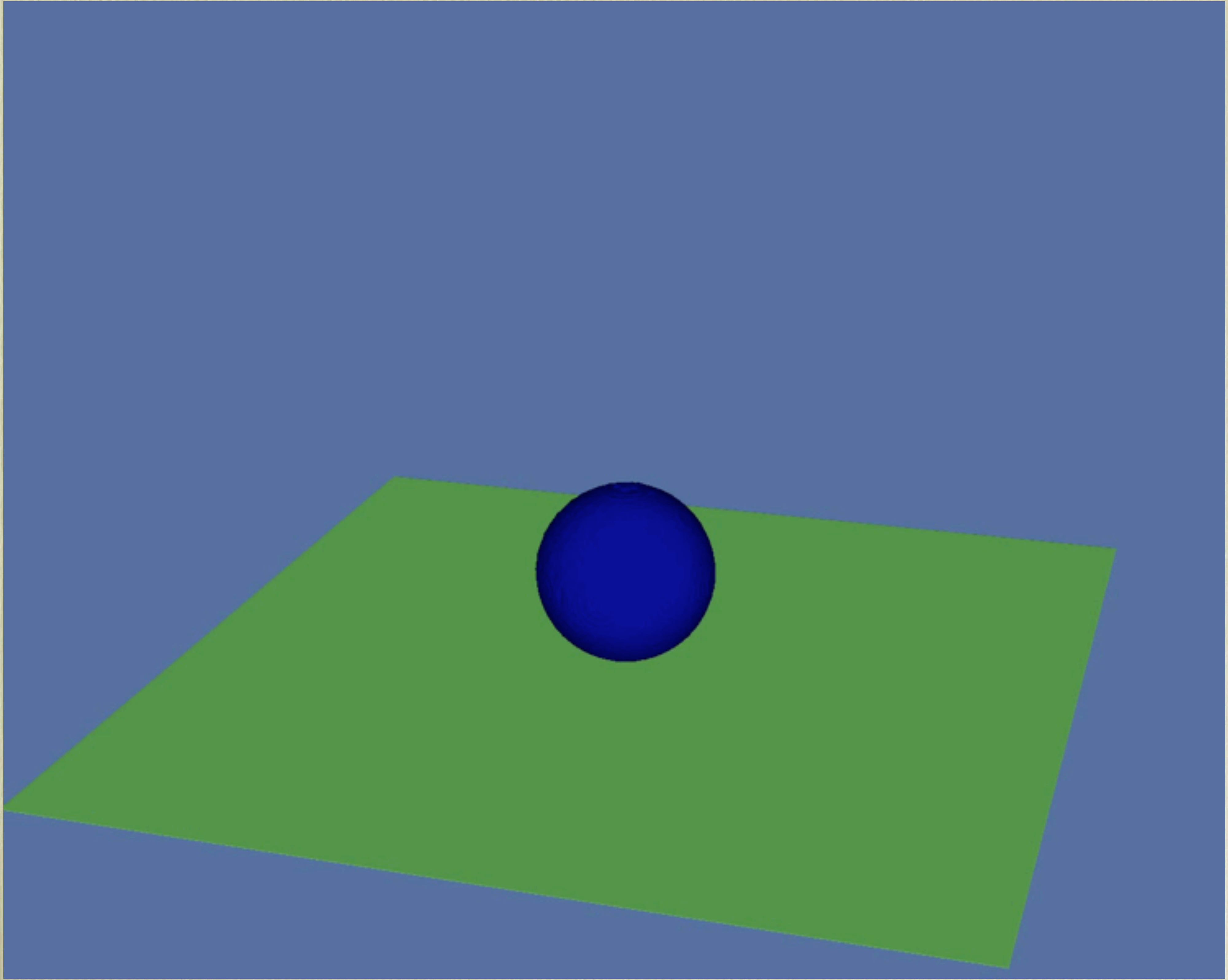

$max\,V\,Re\ =\ 2000;\ \rho f/\rho g = 826;\ Scaling\ Rg\,gaz$
Rg = 2139.2
Rg = 1069.6
Rg = 534.8
Rg = 267.4
Rg = 133.7
Rg = 53.5
0.2
0.3
0.4
0.5
time t*U/D

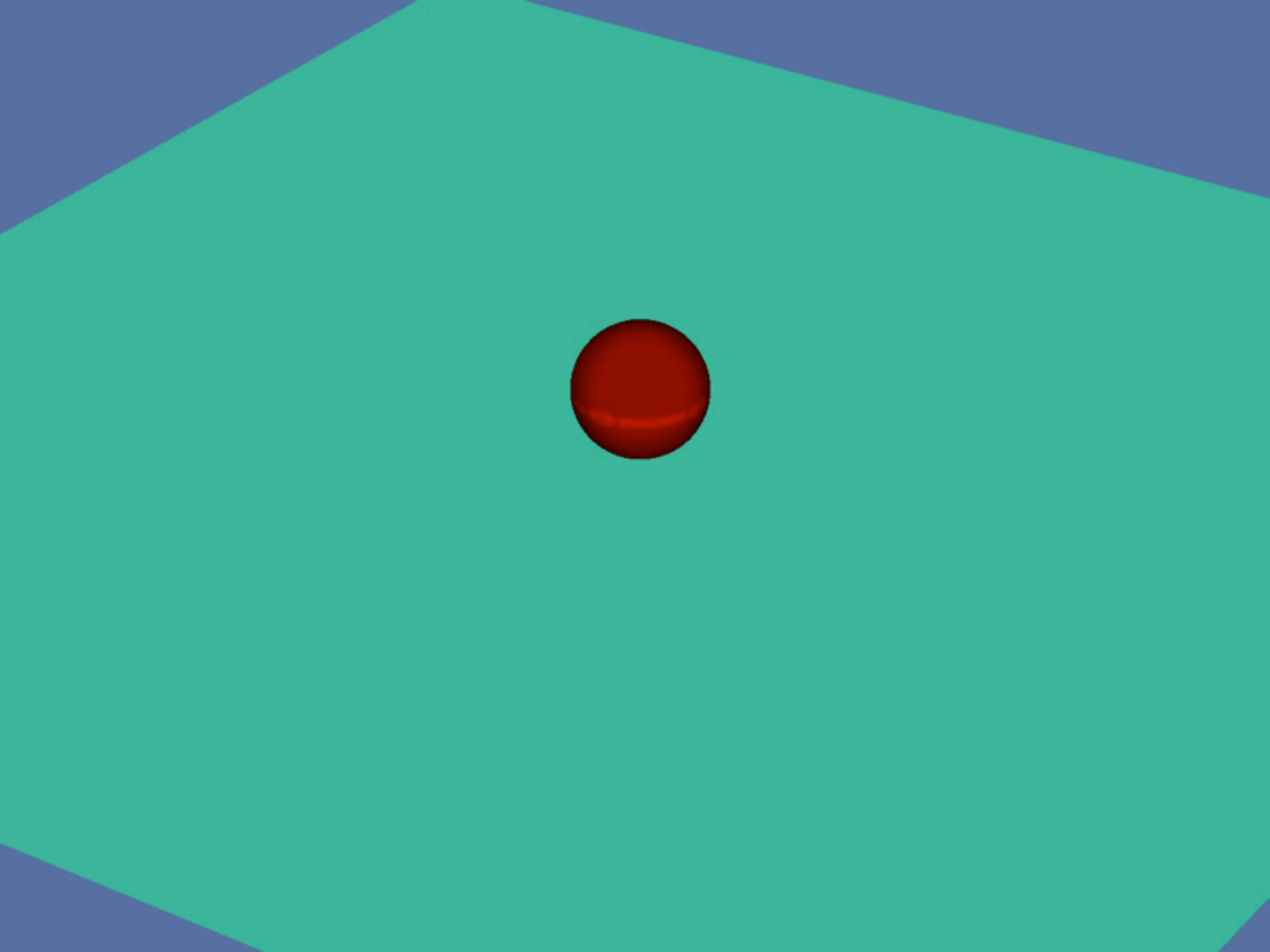

time jet formation
1/Re
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.0005
0.001
0.0015
0.002
0.0025


time jet formation
1/Reg^2/3
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08



St=5.66 10^{-7}
St=1.13 10^{-6}
St=2.26 10^{-6}
St=4.53 10^{-6}
St=9.05 10^{-6}
St=2.26 10^{-5}
relation (4.9)
100
10
10
100
Re t U0/D
St^{-2/3} t U0/D

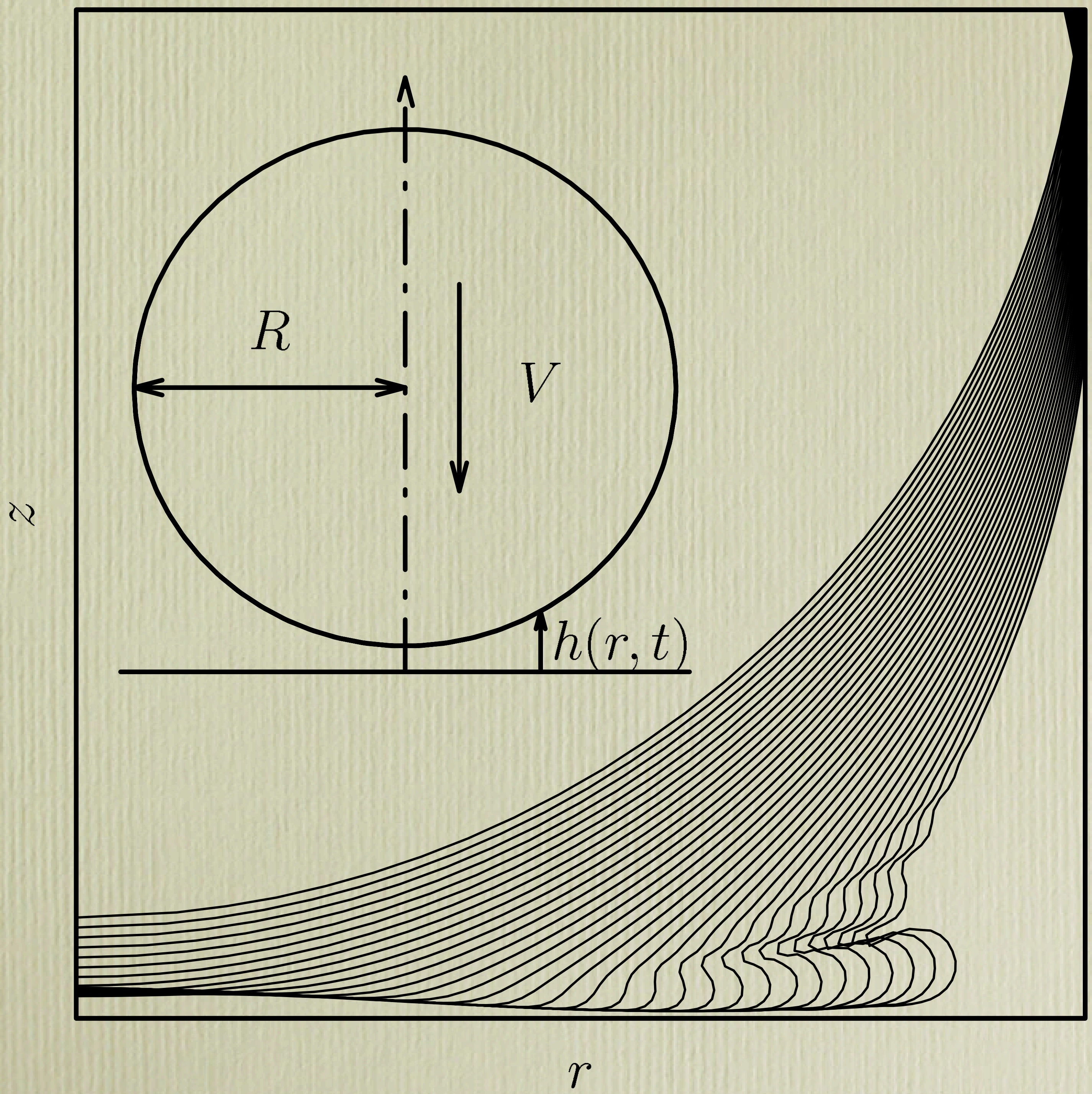




Surrounding gas effects?

- compressibility?
- bubble entrapment
- moving contact line-air entrapment
- aerodynamical instability

- can the initial liquid-solid contact trigger the splash?
- simplified model coupling inviscid flow in the drop with lubrication equation in the gas (S. Mandre, M. Mani, and M. P. Brenner. Precursors to splashing of liquid droplets on a solid surface. Phys. Rev. Lett., 102, 2009).
- In this 2D version, a finite time singularity is observed in the no surface tension case. With surface tension, the singularity disappears and the capillary waves look as precursors of the jets.
- Problem: alone, it cannot explain the experiment!



R
V
h(r,t)
z
r

Model: set of equations

$$\vec{U} = (u_r, u_z) = \vec{\nabla}\varphi(r, z, t)$$

$$\Delta\varphi = 0$$

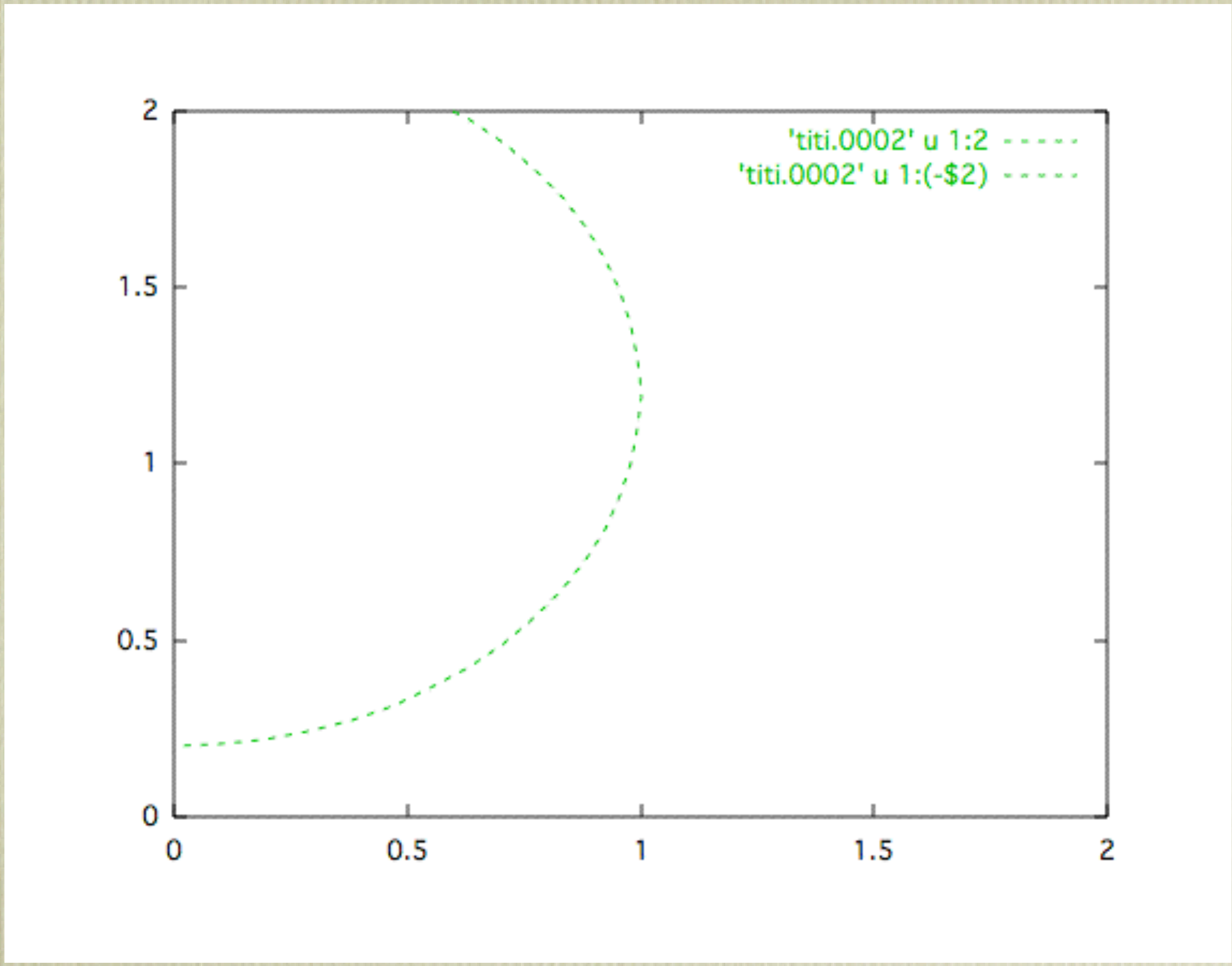
$$\frac{\partial h}{\partial t} = \frac{\partial \varphi}{\partial z} - \frac{\partial \varphi}{\partial r}\frac{\partial h}{\partial r},$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2}\nabla\varphi^2 + p + \frac{1}{We}\kappa = 0$$

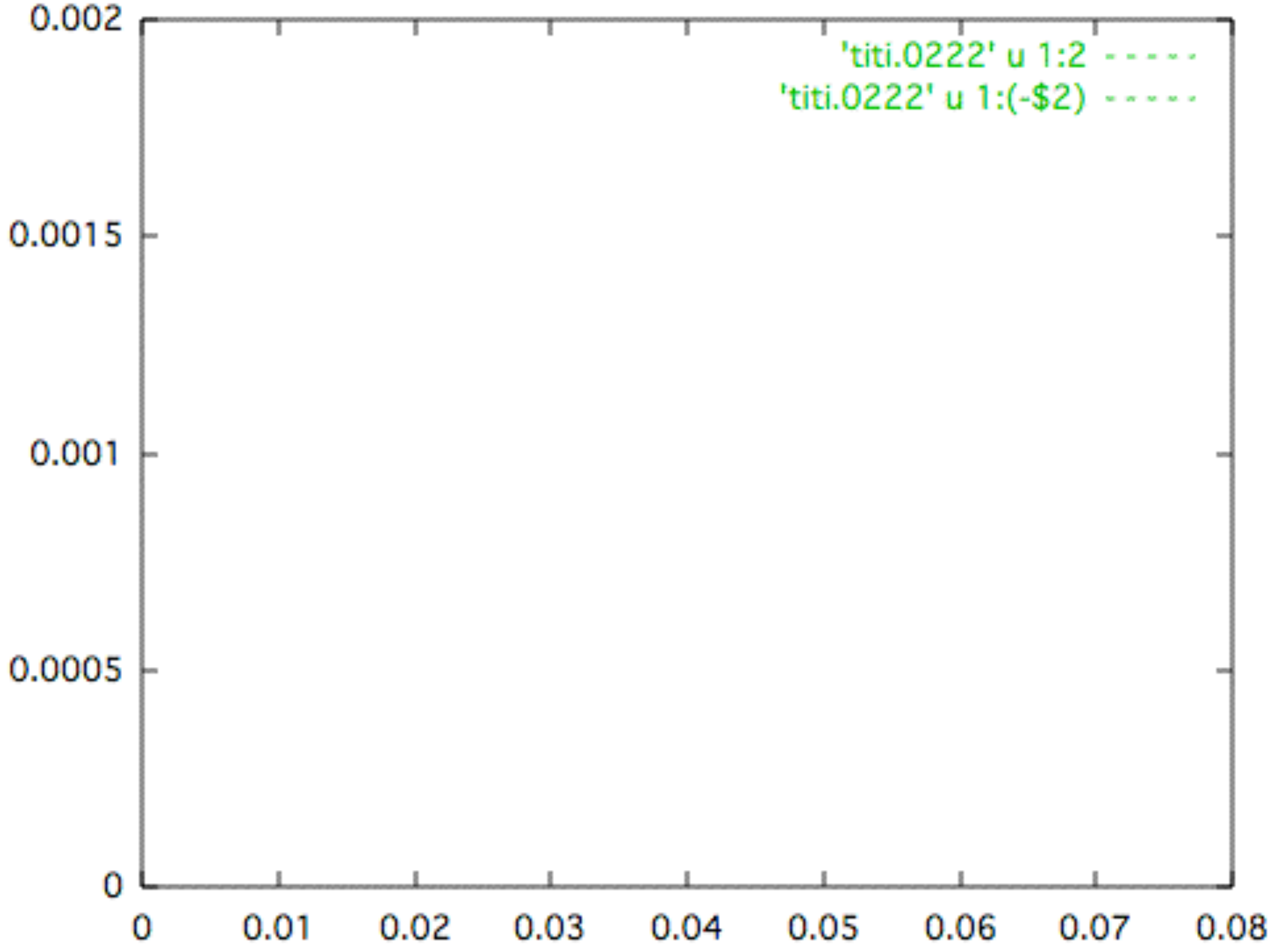
$$\frac{\partial h}{\partial t} = \frac{1}{12 r St}\frac{\partial}{\partial r}\left(r h^3 \frac{\partial p}{\partial r}\right)$$

$$We = \frac{\rho U^2 D}{\sigma}$$

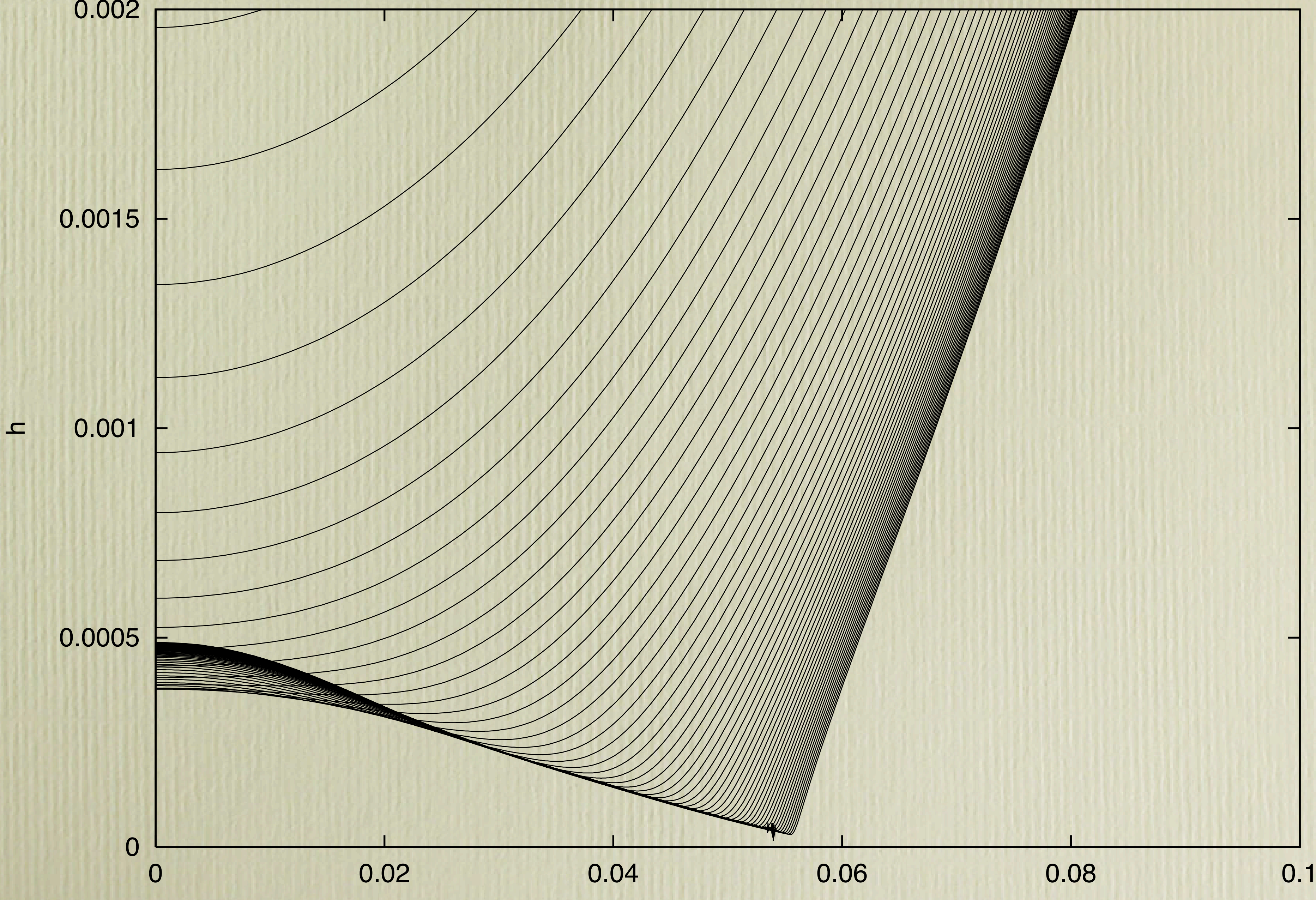
$$St = \frac{\eta}{\rho V R}$$



'titi.0002' u 1:2
'titi.0002' u 1:(-$2)
0
0.5
1
1.5
2

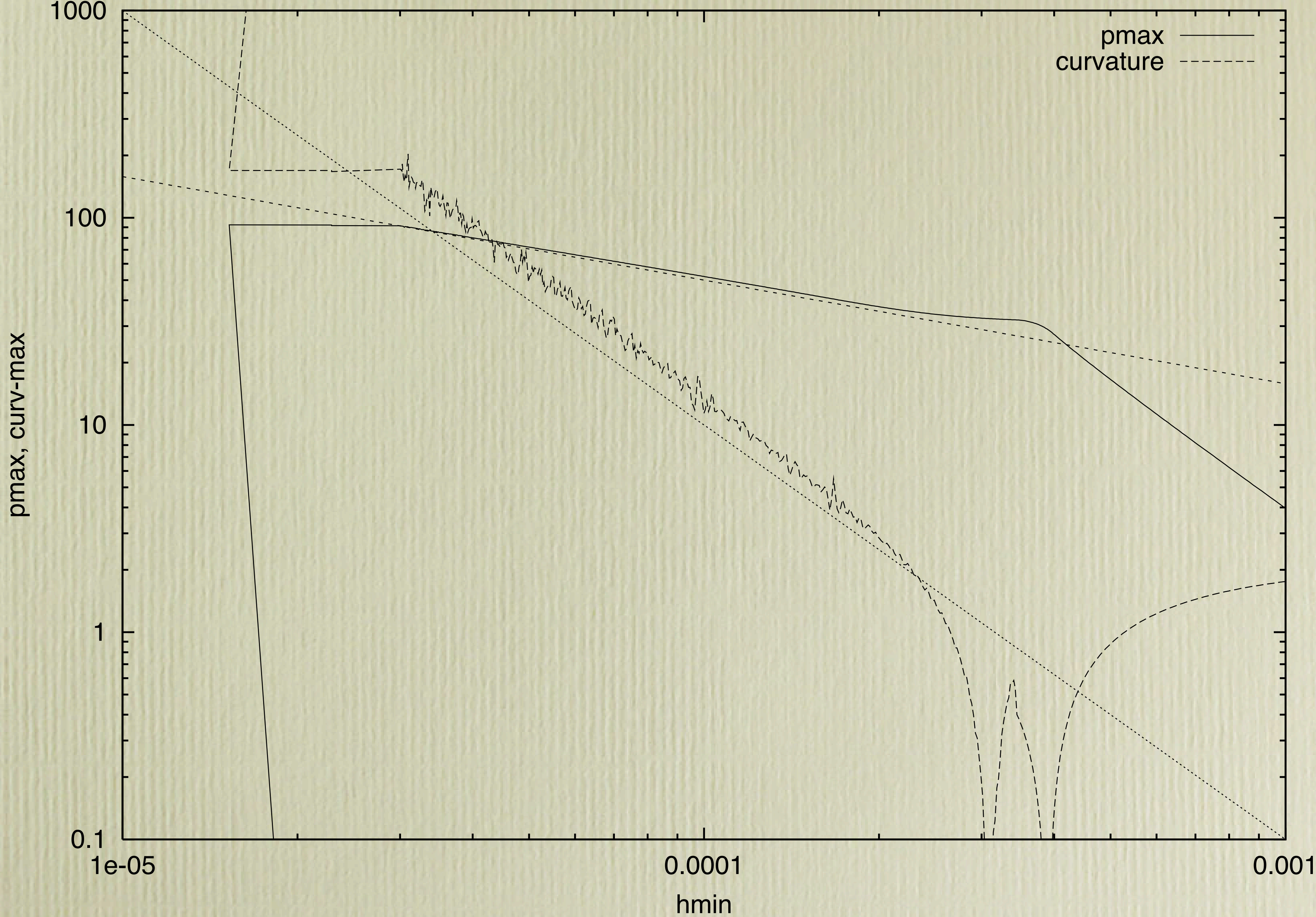

0.002
0.0015
0.001
0.0005
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
'titi.0222' u 1:2
'titi.0222' u 1:(-$2)

No surface tension: finite time singularity



Singuarity properties

- minimal air gap hmin(t) vanishes
- maximal pressure Pmax(t) diverges
- maximal curvature Kmax diverges
- Pmax and hmin are not at the same location although they merge at the singularity
- the peak moves at a constant radial velocity



$$p_{max} \propto h_{min}^{-\frac{1}{2}} \qquad \kappa_{max} \propto h_{min}^{-2}$$

Singularity: self-similar analysis

$$p(r,t) = p_0(t)P(R) \qquad h(r,t) = h_0(t)H(R)$$

$$\varphi(r,z,t) = \varphi_0(t)\Phi(R,Z)$$

Moving frame: $$R = \frac{r - r_0(t)}{l(t)}$$

$$Z = \frac{z}{l(t)}$$

$$(\partial\Omega) - \frac{\varphi_0 \dot{r}_0}{l}\partial_R \Phi + \frac{1}{2}\frac{\varphi_0^2}{l^2}\nabla\Phi^2 + p_0 P = C(t),$$

$$(\partial\Omega) \qquad - \frac{h_0 \dot{r}_0}{l}H' = \frac{h_0^3 p_0 (H^3 P')'}{12\ \mathrm{St}\ l^2},$$

$$(\partial\Omega) - \frac{h_0 \dot{r}_0}{l}H' = \frac{\varphi_0}{l}(\partial_Z \Phi - \frac{h_0}{l}H'\partial_R \Phi),$$

$$(\Omega) \qquad \Delta\Phi = 0,$$

2 regimes

Crossover for $h_0 \sim St^{4/3}$

$$h_0(t) \ll l(t)$$

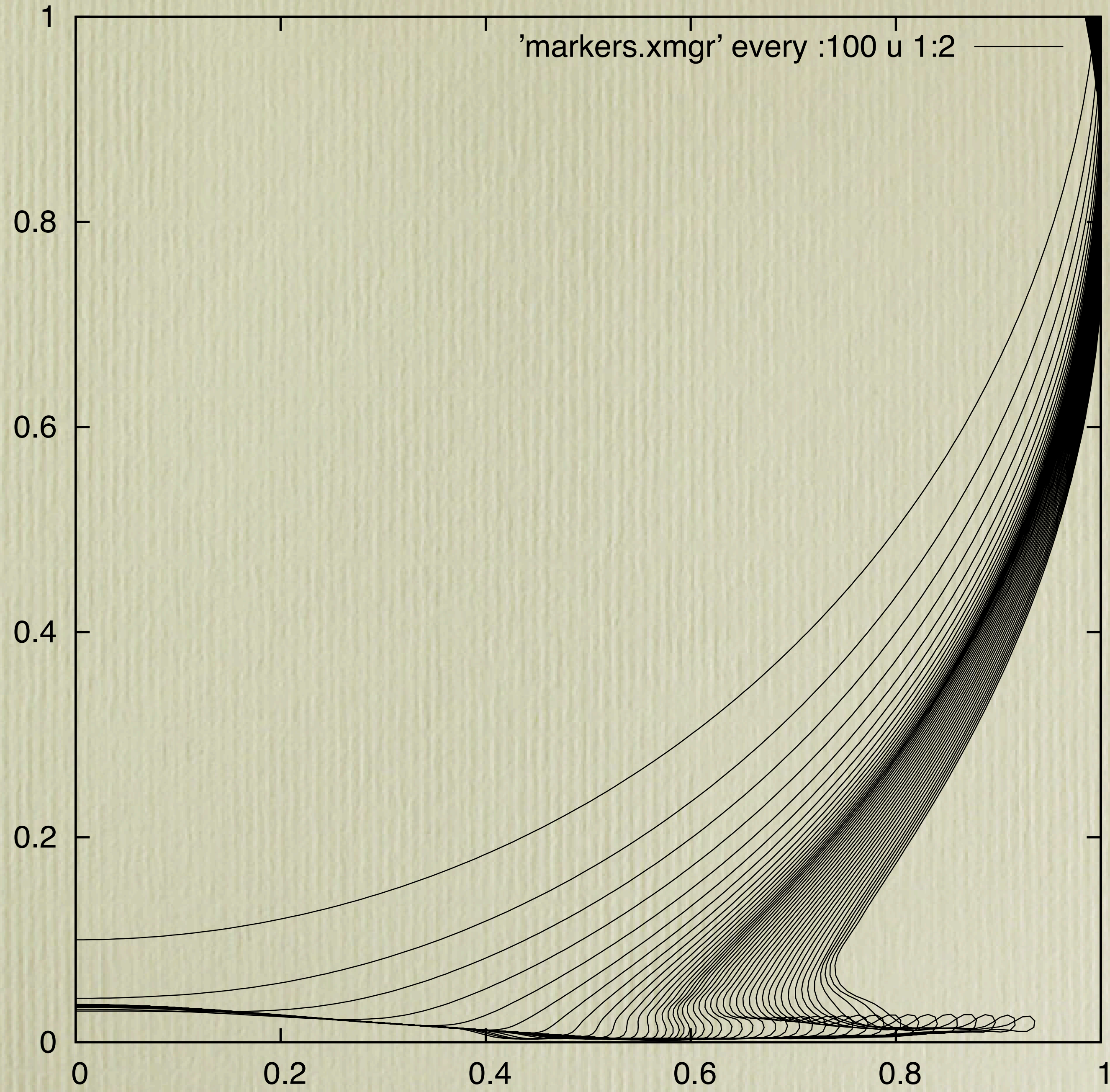
$$p_0 \sim h_0^{-1/2} \text{ and } \kappa_0 \sim \mathrm{St}^{4/3} h_0^{-2}.$$

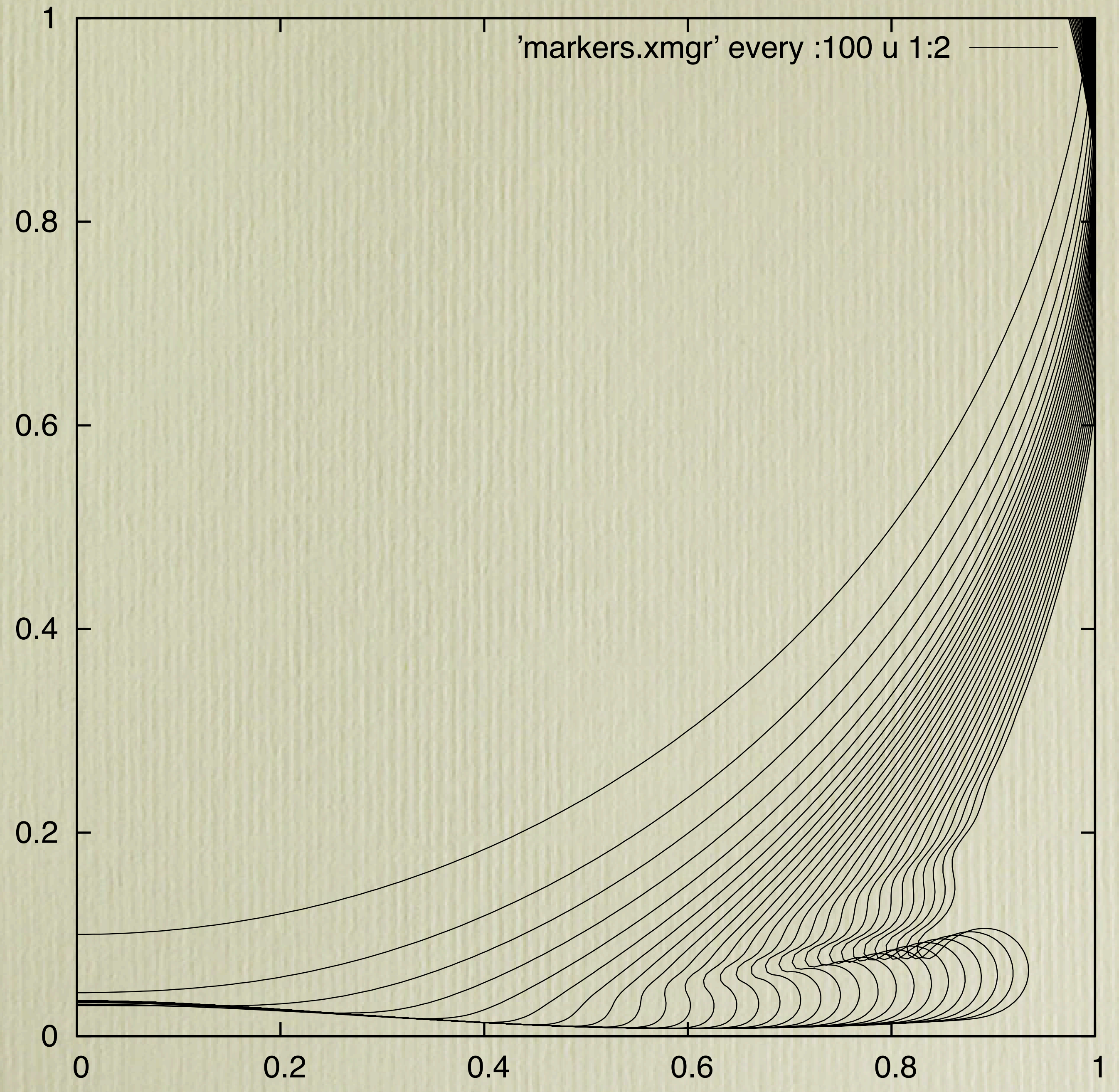
$$l(t) \ll h_0(t)$$

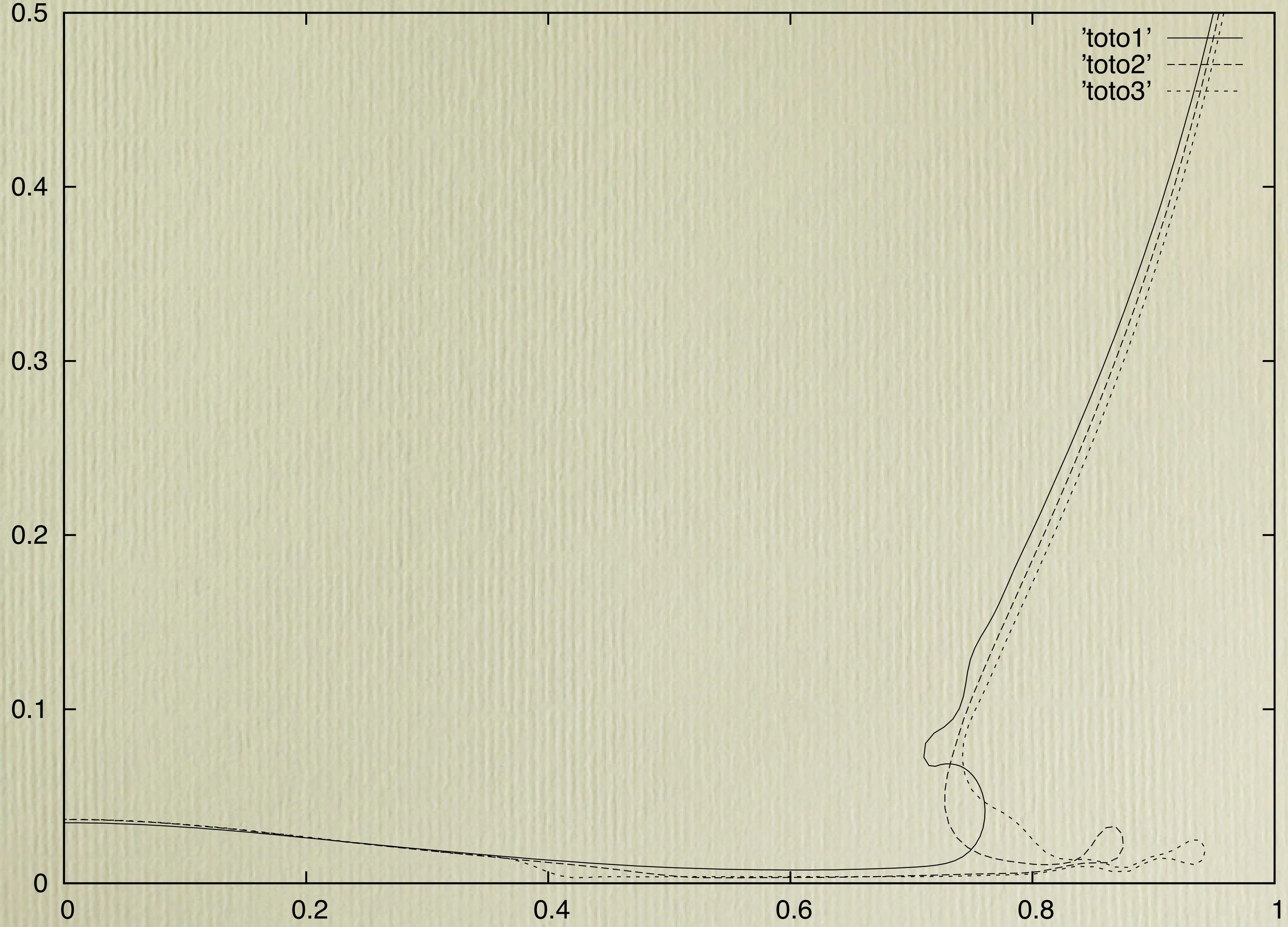
$$p_0 \sim \mathrm{St}^{-2/3} \text{ and } \kappa_0 \sim \mathrm{St}^{8/3} h_0^{-3}.$$

Effect of surface tension

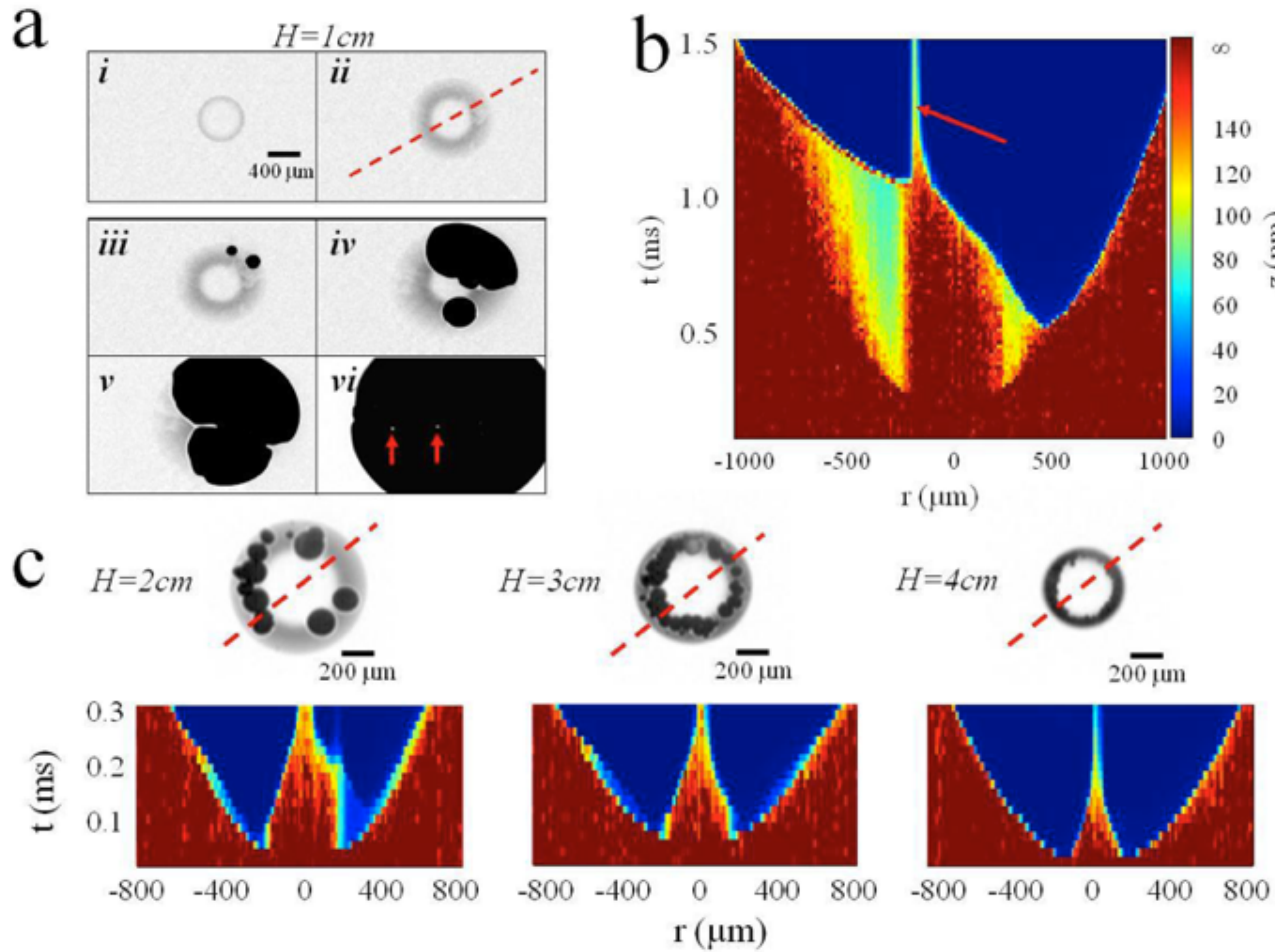
- should «regularize» the singularity
- capillary waves
- does the liquid «touch» the substrate?
- viscous film pinch-off: no finite time singularity!
- liquid viscosity


'markers.xmgr' every :100 u 1:2
0
0.2
0.4
0.6
0.8
1

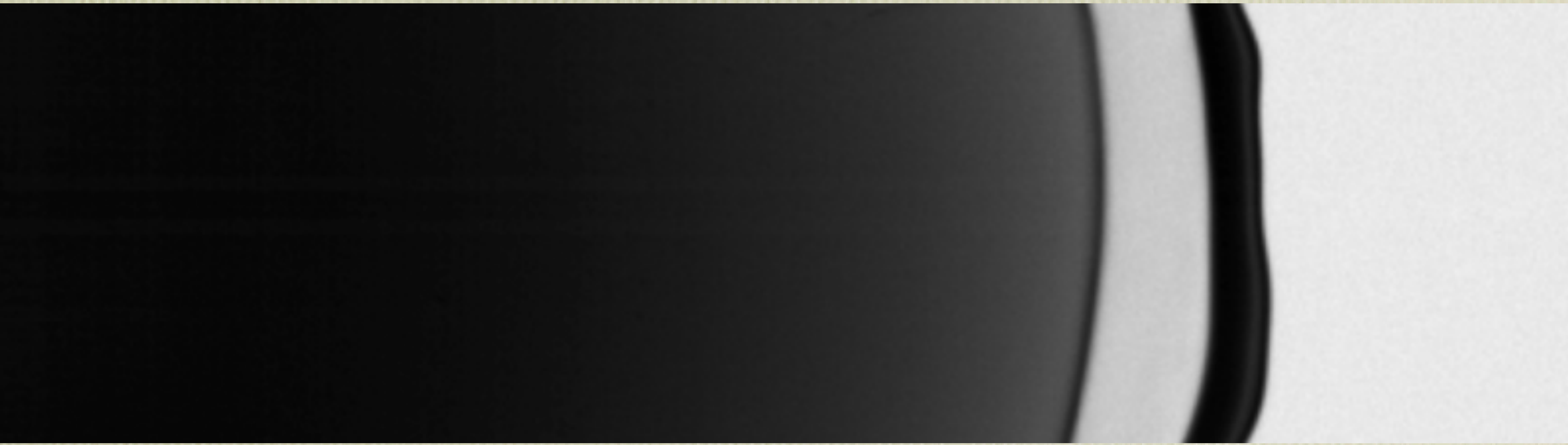

'markers.xmgr' every :100 u 1:2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1



'toto1'
'toto2'
'toto3'
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1



Kolinski et al, 2011



Can we explain the air influence on the impact?

- lubricated gas with inviscid liquid should not be enough!
- compressibility? (Mani,Mandre & Brenner 2009,2010, 2012)
- surface forces?
- 1-rarefied gas limit?
- 2-gas inertia?