SIMULATION OF A 2D GRANULAR COLUMN COLLAPSE ON A RIGID BED WITH LATERAL FRICTIONAL EFFECTS

High slope results and comparison with experimental data

Nathan Martin[1], Ioan Ionescu[2], Anne Mangeney[1,3]
François Bouchut[4], Olivier Roche[5] and Maxime Farin[1]

1 Institut de Physique du Globe de Paris, Seismology Team
2 *LSPM*, University Paris-Nord, Sorbonne Paris Cité, France
3 *ANGE INRIA Team* - Jacques Louis Lions - CETMEF
4 *LAMA*, University Paris-Est, CNRS, UPEM, UPEC
5 Laboratoire Magmas et Volcans, University Blaise Pascal-CNRS-IRD

Projet IdEx PEGES, projet ERC SLIDEQUAKES

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Introduction

- Landslides are *unique unpredictable* events

- *Project goal* : use *seismic signals* and numerical modelling to infer and constrain parameters

- Granular material are treated as *nonlinear viscoplastic* fluid

  Requires specific numerical tools to deal with

  non linearity and non differentiability

- The column collapse is a highly transient problem with unitary aspect ratio (at the beginning of the collapse)

  Navier-Stokes equation to capture all the physics

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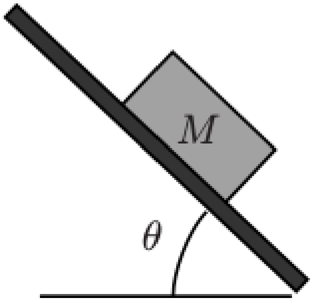
Mechanical model

- Navier-Stokes for momentum equilibrium

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right) - \mathbf{div}\,\boldsymbol{\sigma}' + \nabla p = \rho\boldsymbol{f}$$

- How to describe the mechanical behavior of a granular material?
  - Basic approach : **plasticity**

=> Describe a flow/no flow behavior through a **plasticity criterion** :



The block on the ramp : *friction coefficient* $= \tan(\delta)$

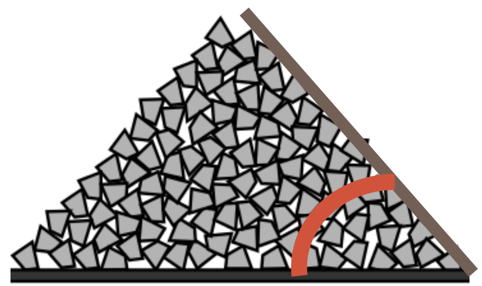
The block starts to slide when

$$mg\sin(\theta) = \tan(\delta)\,mg\cos(\theta)$$

i.e. when $\delta = \theta$

Mechanical model

- For a granular material the *internal friction angle* is related to the maximal angle of a pile :



The *criterion of stability* of a granular material is a *frictional criterion* :

- under a given threshold the medium is rigid
- over it, it deforms and the shear stress is given by $\tau = \mu\sigma$

  where $\mu = \tan(\delta)$ is the friction coefficient

Normal stress

Shear stress

Perfect plasticity :

Infinite strain over a given stress : **rupture**

Mechanical model

For the granular material we are still lacking of a « **flow law** » describing the deformation

- A constitutive law expresses the deviatoric stress as a function the strain rate. Two assumptions are made here :
  1) *Colinerarity* of the deviatoric stress tensor and the strain rate tensor : $\dfrac{\boldsymbol{D}}{||\boldsymbol{D}||} = \dfrac{\boldsymbol{S}}{||\boldsymbol{S}||}$

     Recall : $\boldsymbol{S} = \boldsymbol{\sigma} - p\boldsymbol{Id}$
     $\boldsymbol{D} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}})$
  2) *Incompressibility* : $tr(\boldsymbol{D}) = 0$

We thus obtain : $\boldsymbol{D} = \lambda\,\boldsymbol{S}$ with $\boldsymbol{\lambda} = ||\mathbf{D}||/||\mathbf{S}||$

The simple 1D perfect plasticity criterion is generalized in 2D with a law of the form $||\mathbf{S}|| = \kappa(p)$

Mechanical model

- Drucker-Prager plasticity criterion

$$\kappa(p) = \kappa_0 + \mu_s p$$

Cohesion (set to 0 here)   Internal friction angle

- If we invert the constitutive law using the plasticity criterion we finally obtain (setting cohesion to 0) :

$$\boldsymbol{S} = \mu p \, \frac{\boldsymbol{D}}{||\boldsymbol{D}||}$$

This is a *plastic flow law.* Similarly to a viscous fluid law, it expresses the deviatoric stresses as a function of the strain rates.

The term $\mu p / ||\boldsymbol{D}||$ can be seen as an *effective viscosity* of the material depending on shear rate and pressure

Mechanical model

- The $\mu(I)$ rheology : instead of considering a constant friction angle, the idea is to derive a phenomenological law describing a spatial variability of μ :

$$\boldsymbol{S} = \mu(I)p\ \frac{\boldsymbol{D}}{||\boldsymbol{D}||}$$

- We introduce for that the inertial number I[1,2]:

$$I = \frac{2||\boldsymbol{D}||d}{\sqrt{p/\rho_s}}.$$

Pressure

Grain density

Grain diameter

(It can be seen as a ratio between the microscopic *particle rearrangement timescale* $d/\sqrt{p/\rho_s}$ and macroscopic *strain rate timescale* $1/||\boldsymbol{D}||$)

1 : S.B. Savage, The mechanics of rapid granular flows, Adv. Appl. Mech., 1984
2 : C. Ancey & al, Framework for concentrated granular suspensions in steady simple shear flow, J. Rheol., 1999

Mechanical model

- The $\mu(I)$ rheology[3] is written :

internal friction angle at high I

$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{1 + I_0/I}$$

internal friction angle at low I

- If we develop the constitutive law using the definition of $\mu(I)$ and I :

$$\boldsymbol{S} = p\,\frac{\boldsymbol{D}}{||\boldsymbol{D}||}\left(\mu_s + \frac{\mu_2 - \mu_s}{\frac{I_0\sqrt{(p/\rho)}}{||\boldsymbol{D}||d} + 1}\right)$$

3: P. Jop, Y. Forterre, O. Pouliquen, A constitutive law for dense granular flows, Nature, 2006

Mechanical model

which is finally written (setting $k = d\sqrt{\rho}$) :

$$\boldsymbol{S} = \mu_s p \frac{\boldsymbol{D}}{||\boldsymbol{D}||} + \boxed{\frac{(\mu_2 - \mu_s)p}{||\boldsymbol{D}|| + \frac{I_0}{k}\sqrt{\mathrm{p}}}\boldsymbol{D}}$$

We retrieve the original **plastic term**

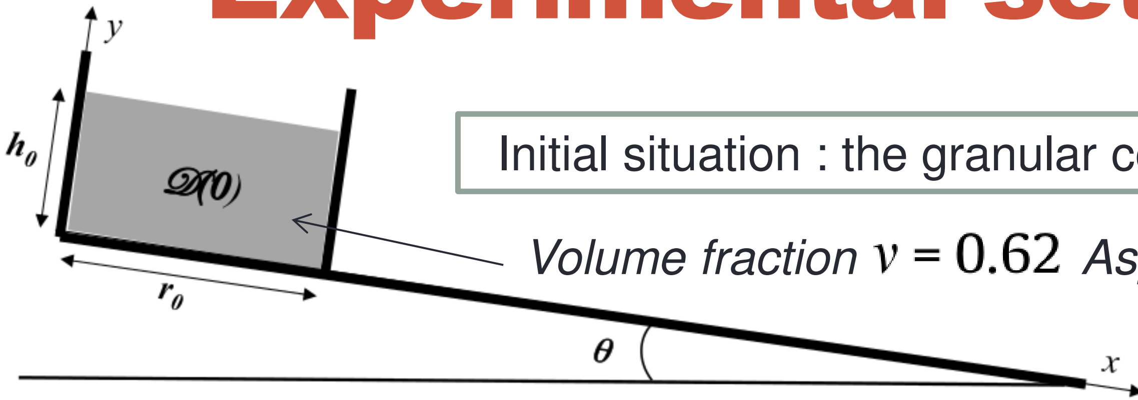
plus an additional **viscous term** introducing a viscosity

$$\eta = \frac{1}{2}\frac{(\mu_2 - \mu_s)p}{||\boldsymbol{D}|| + \frac{I_0}{k}\sqrt{\mathrm{p}}}$$

The model is now describing a **viscoplastic fluid** flow.

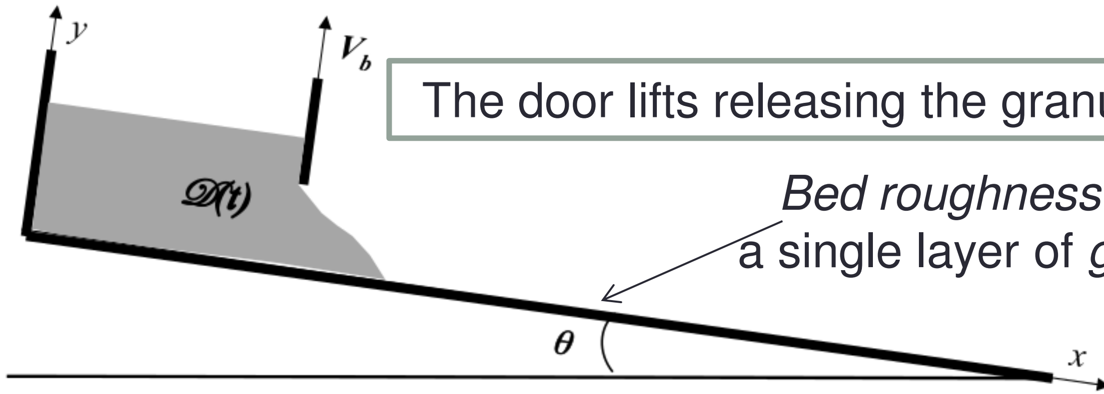
- 2 viscous flow laws are then considered :
  - Constant viscosity η ⇒ *Drucker-Prager fluid*
  - Pressure-dependent viscosity $\eta(\|\boldsymbol{D}\|, p)$ ⇒ $\mu(I)$ *rheology*

Experimental setup[4,5]



Initial situation : the granular column lies behind a door

Volume fraction $\nu = 0.62$ *Aspect ratio* $h_0/r_0 = 0.7$



The door lifts releasing the granular column on the channel

Bed roughness simulated with a single layer of *glued glass beads*

Boundary conditions

- *Left wall and bed* : Coulomb friction with coeff $\mu_C = \begin{cases} \mu_w \text{ on left wall (plexiglas)} \\ \mu_s \text{ at bottom (glass beads layer)} \end{cases}$
- *Uplifting door* : free slip

Surface : *free surface moving in time through a transport equation*

4 : A. Mangeney & al., Erosion and mobility in granular collapse over sloping beds, JGR – Earth 2010
5 : I. Ionescu & al, Viscoplastic modeling of granular column collapse, JNNFM, 2015

Accounting for lateral friction

- We consider a 3D domain $\mathcal{D}(t) = \Omega(t) \times \left]\frac{-w}{2}, \frac{w}{2}\right[$ with a solution constant in transverse direction
- The boundary term $\displaystyle\int_{\Omega(t)} \boldsymbol{\sigma} n \cdot \varphi \, dx$ is equal to

$$\underbrace{w \int_{\Gamma_b} \boldsymbol{\sigma} n \cdot \varphi \, ds}_{\text{Former boundary term}} + \underbrace{2 \int_{\Omega(t)} \boldsymbol{\sigma} n \cdot \varphi \, ds}_{\text{New « volumic » boundary term}}$$

where $\boldsymbol{\sigma} n = \underbrace{\sigma_n \mathbf{n}}_{\text{Normal stress}} + \underbrace{\boldsymbol{\sigma}_T}_{\text{Tangential stress}}$ and $\boldsymbol{\sigma}_T = \begin{cases} \sigma_{t_1} \mathbf{t_1} + \sigma_{t_2} \mathbf{t_2} \text{ in 3D} \\ \sigma_t \mathbf{t} \text{ in 2D} \end{cases}$

- The rest of the VF is simply multiplied by the width w

Accounting for lateral friction

- The tangential stress is given by a Coulomb friction law of the form :

$$u \cdot n = 0, \begin{cases} \sigma_T = -\mu[-\sigma_n]_+ \frac{u_T}{|u_T|} \\ |\sigma_T| \leq \mu[-\sigma_n]_+ \end{cases}$$

- On the lateral faces, the normal stress σ_n is in the transverse direction z thus equal to the 3D diagonal term of the stress tensor σ_{zz}

- From both 2D and 3D incompressibility, we naturally obtain : $\sigma_{zz} = -p$

Accounting for lateral friction

- The Coulomb friction law leads to write the « volumic » boundary term as : $\displaystyle\int_{\Omega(t)} \boldsymbol{\sigma} n \cdot \varphi \, dx =$

$$\underbrace{\int_{\Omega(t)} -p\mathbf{z} \cdot \varphi \, dx}_{\substack{\textbf{= 0 because no flux} \\ \textbf{in the transverse direction}}} - \int_{\Omega(t)} \mu p \frac{\mathbf{u}}{|\mathbf{u}|} \, dx$$

- If we divide the whole VF by w, we retrieve the former VF plus the extra term :

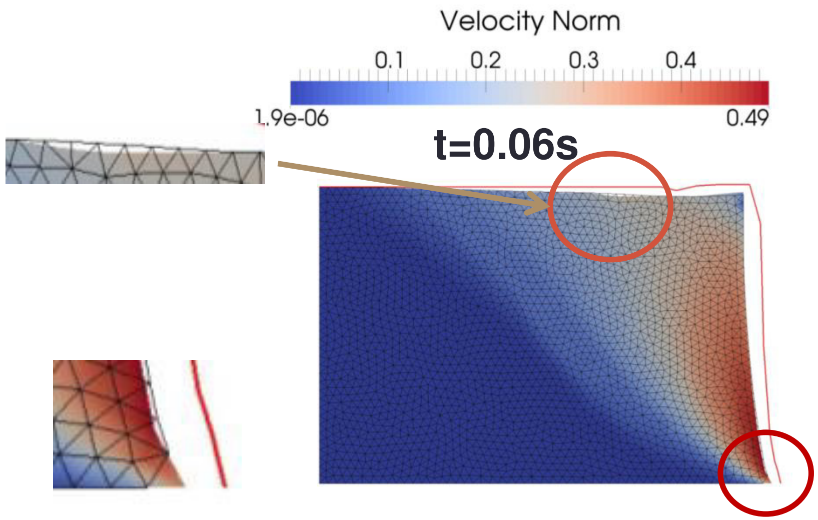
$$\frac{2}{w} \int_{\Omega(t)} \mu p \frac{\mathbf{u} \cdot \varphi}{|\mathbf{u}|}$$

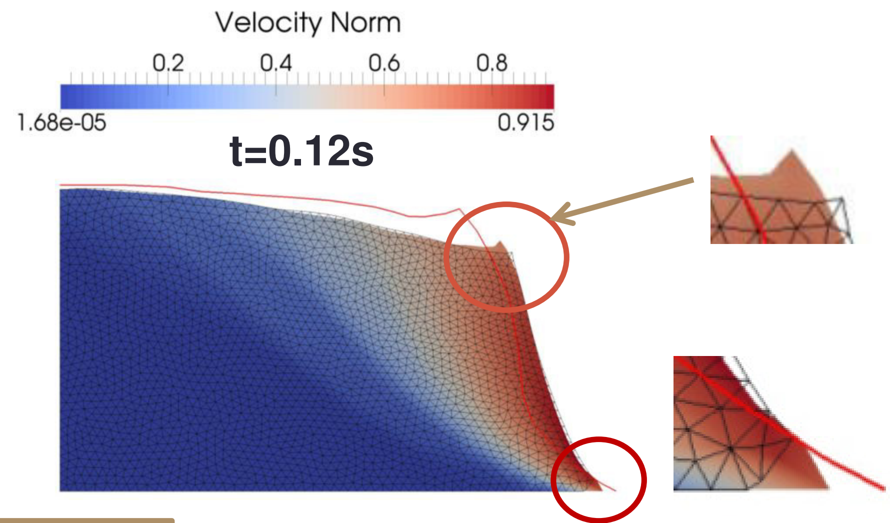
Thickness profile results

- We plot thereafter *computed* and *observed thickness profiles* at various times during the collapse for different slopes (10°,19°,22°)

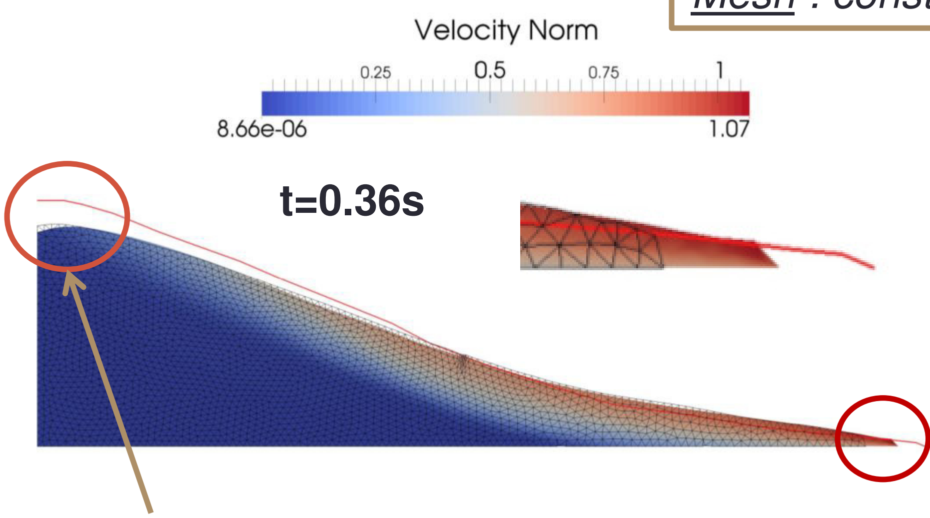
- The results obtained with $\mu(I)$ rheology are compared with the constant viscosity model

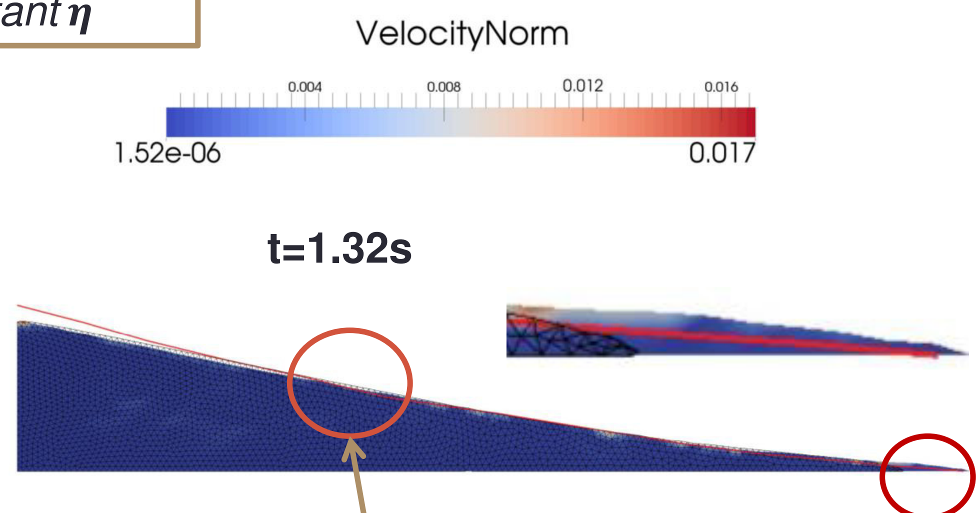
Comparison with constant viscosity : $\alpha = 10°$



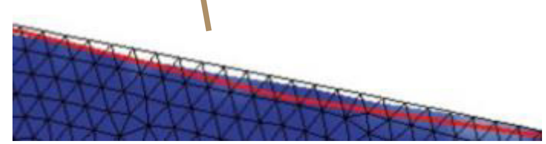


Colored surface : $\mu(I)$
Mesh : constant η

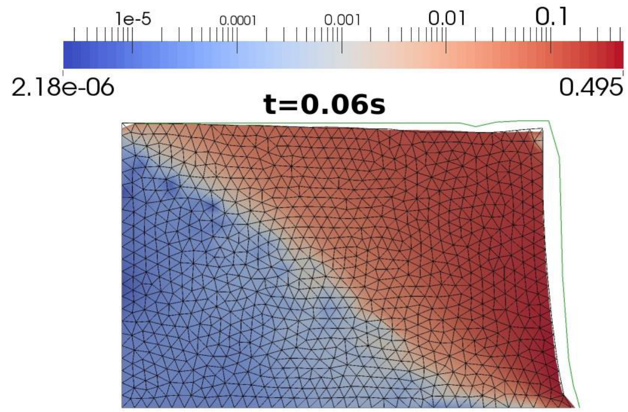


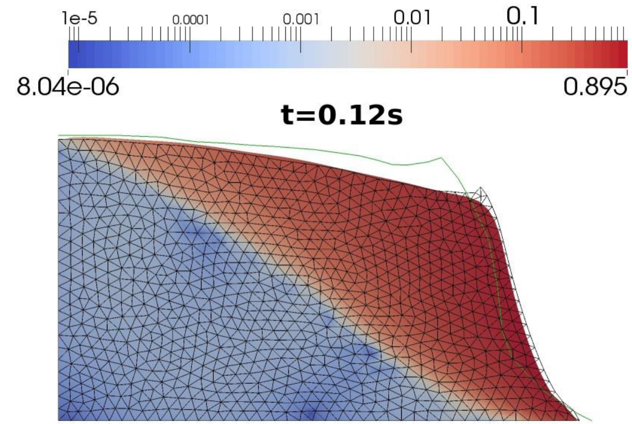


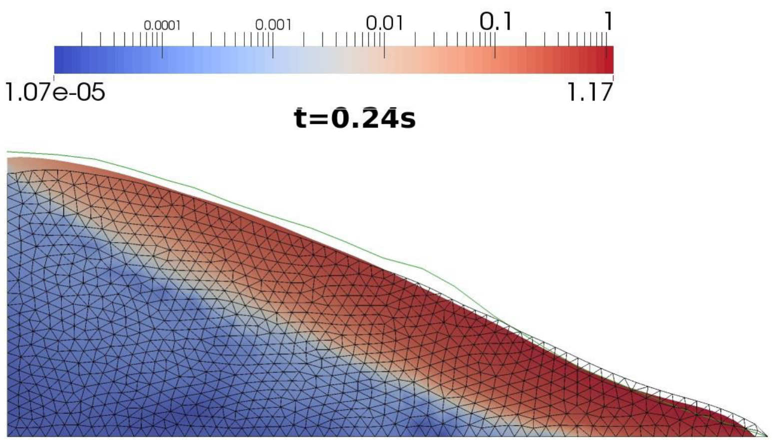
Results do not differ for higher slopes

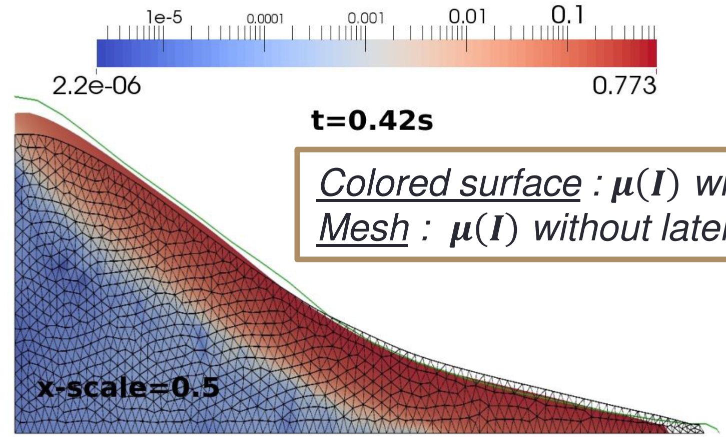


Thickness profile results : $\alpha = 10°$

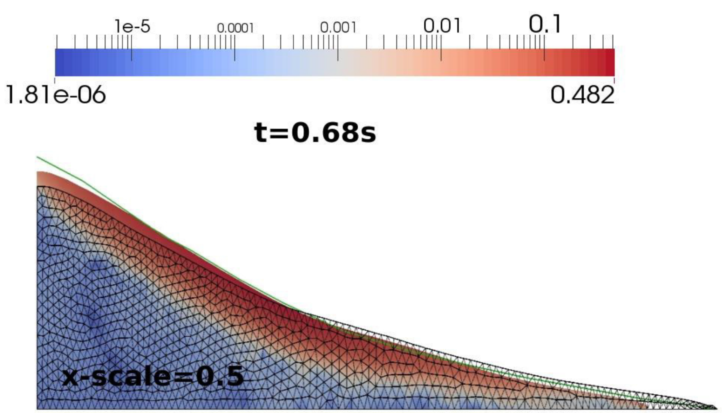


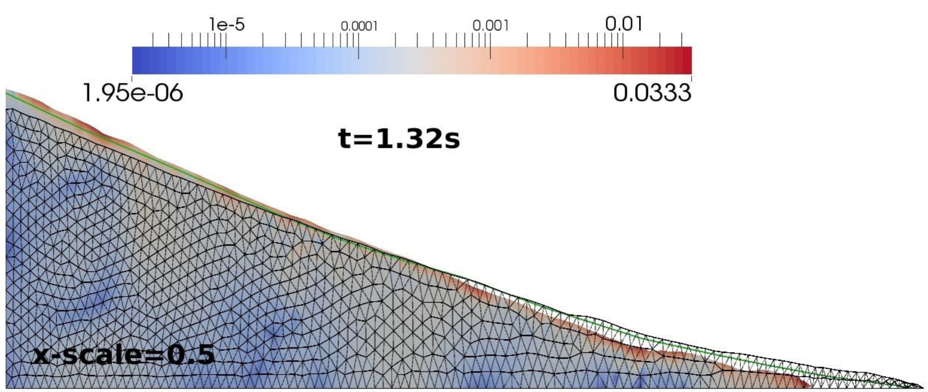




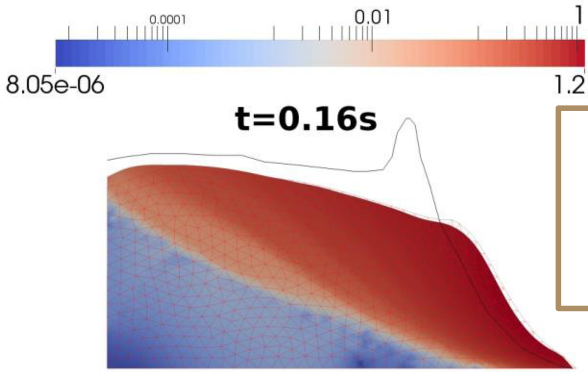


Colored surface : $\mu(I)$ *with lateral friction*
Mesh : $\mu(I)$ *without lateral friction*

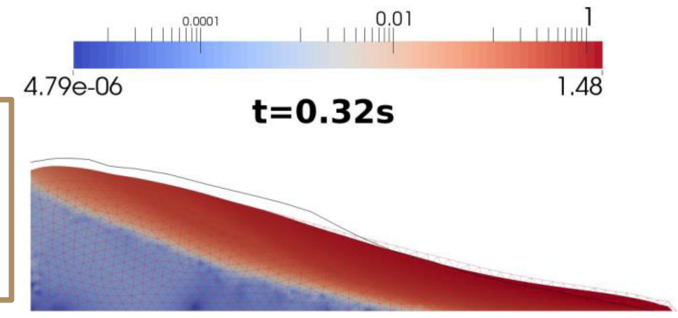


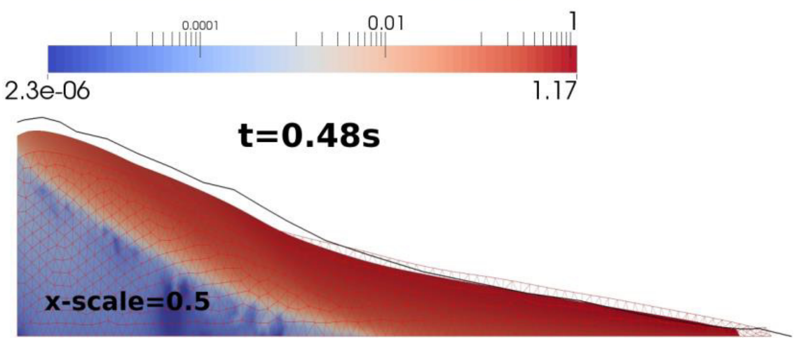


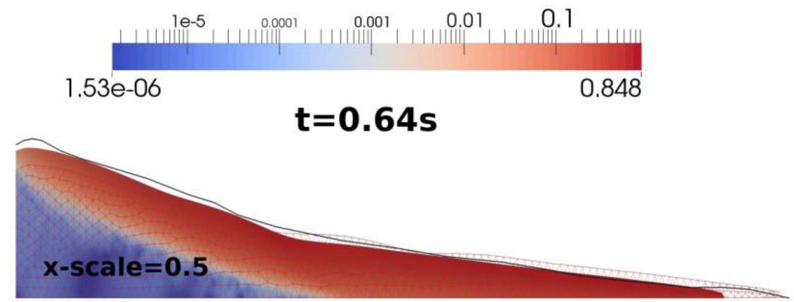
Velocity norm in logarithmic scale

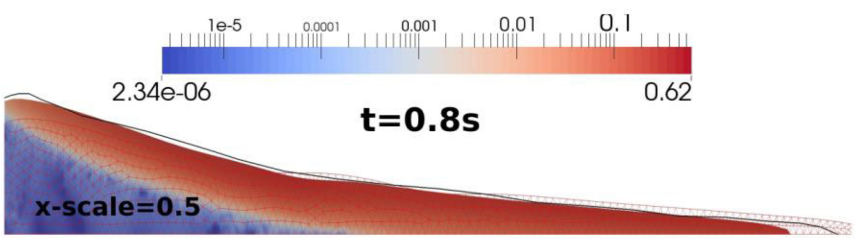


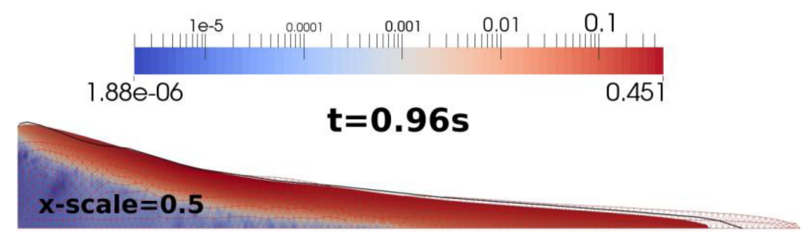
Colored surface : $\mu(I)$ with lateral friction
Mesh : $\mu(I)$ without lateral friction

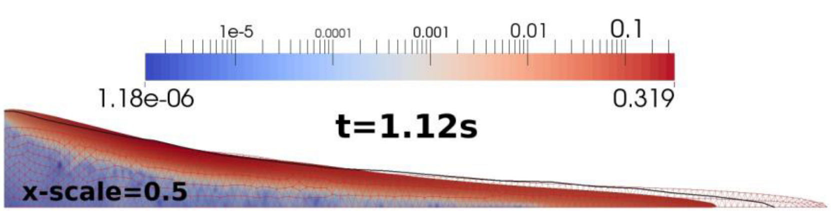


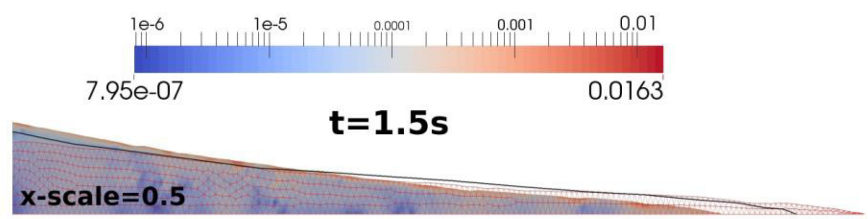






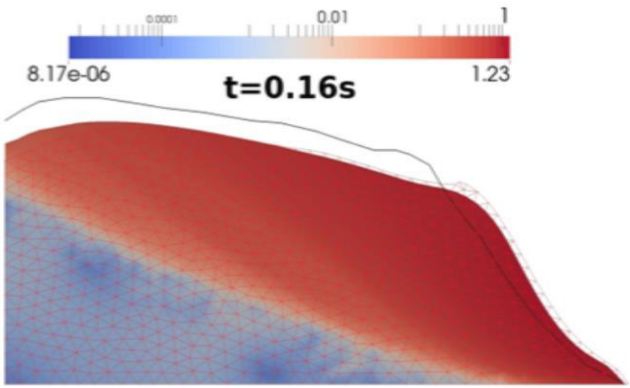


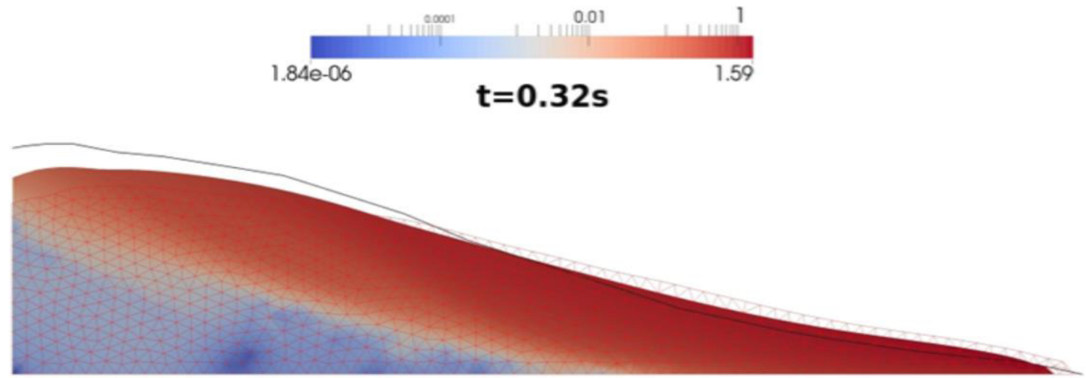


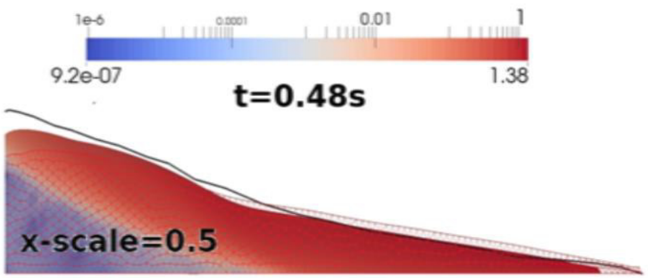


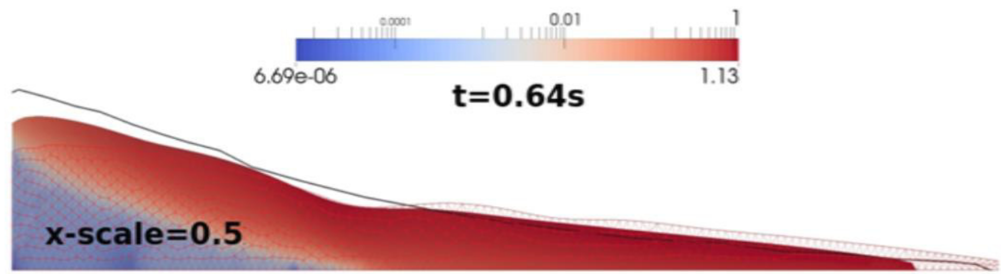
Thickness profile results : $\alpha = 22°$

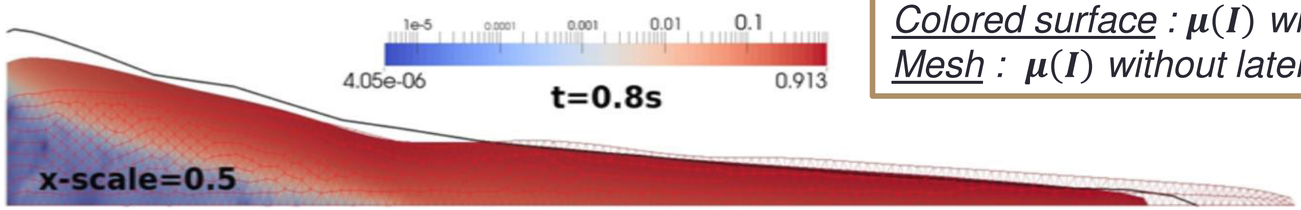
Velocity norm in logarithmic scale



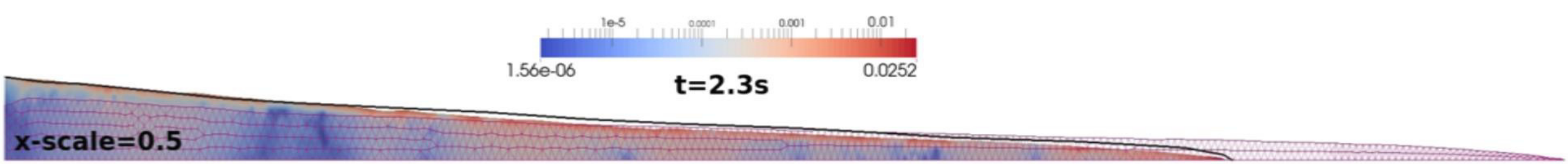








Colored surface : $\mu(I)$ *with lateral friction*
Mesh : $\mu(I)$ *without lateral friction*



Summary

- The model and method reproduce *quantitatively* the flow dynamics
  - The *extent of the deposit* is *very accurate* along the collapse at small to intermediate slopes. At higher slopes, the lateral friction reveals to be necessary for the deposit to stop soon enough
  - The *solid-fluid transition* is sharply determined

  ⇨ « *moves down* » too quickly producing a mass loss on the upper left corner without the lateral friction

  ⇨ the dynamic shape of the deposit is significantly improved with the lateral friction

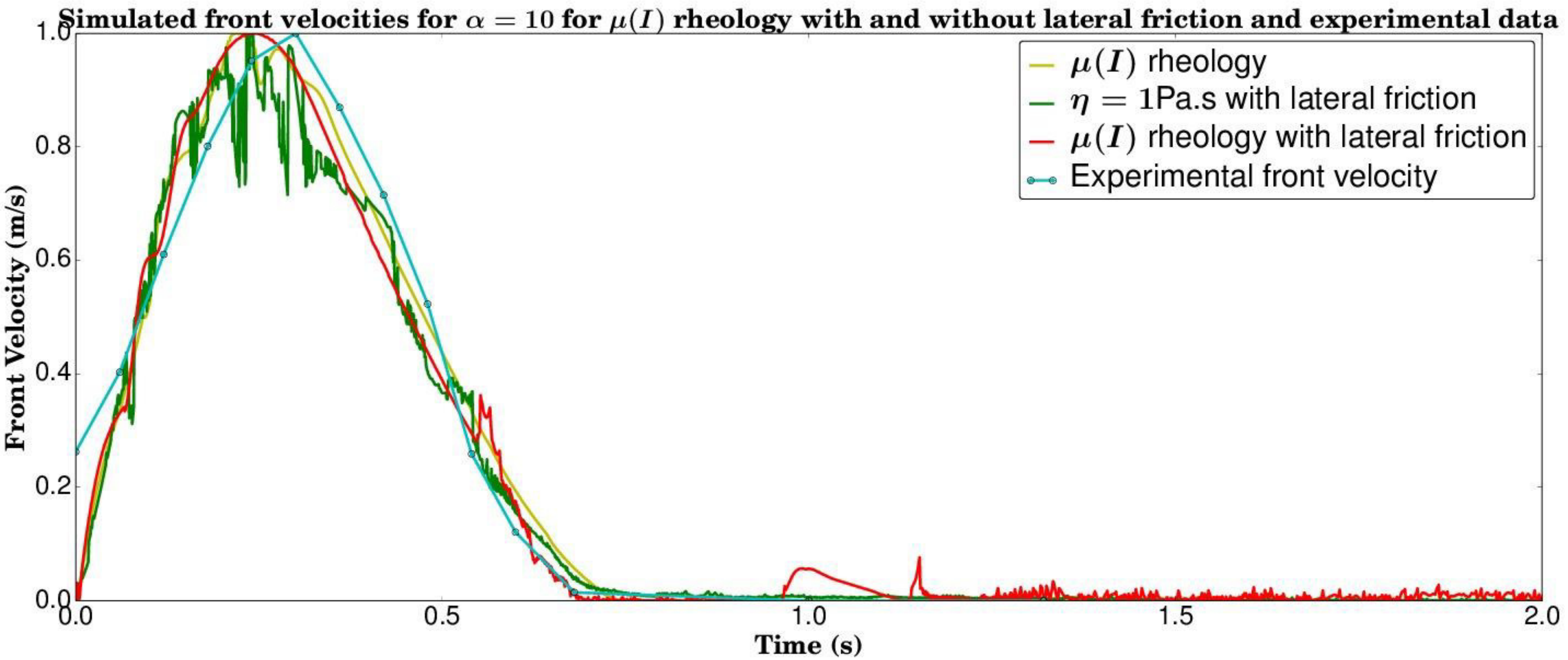
- The rheology $\mu(I)$ has little effect on the flow dynamics

(even where I is high) while being more computationaly extensive.

- The *dilatancy phenomena* on data is strong at the beginning and the incompressible model (obviously) fails to reproduce it

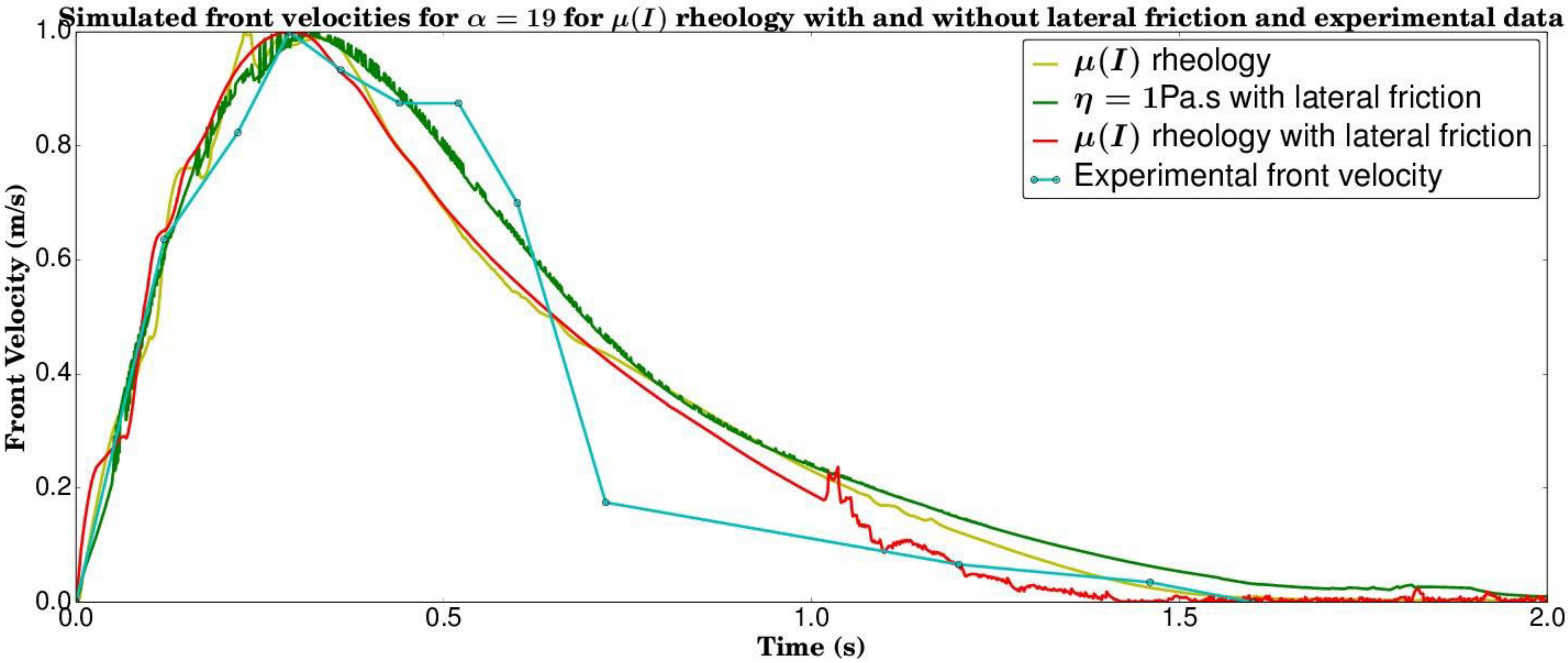
Evolution of front velocities

$$\alpha = 10°$$



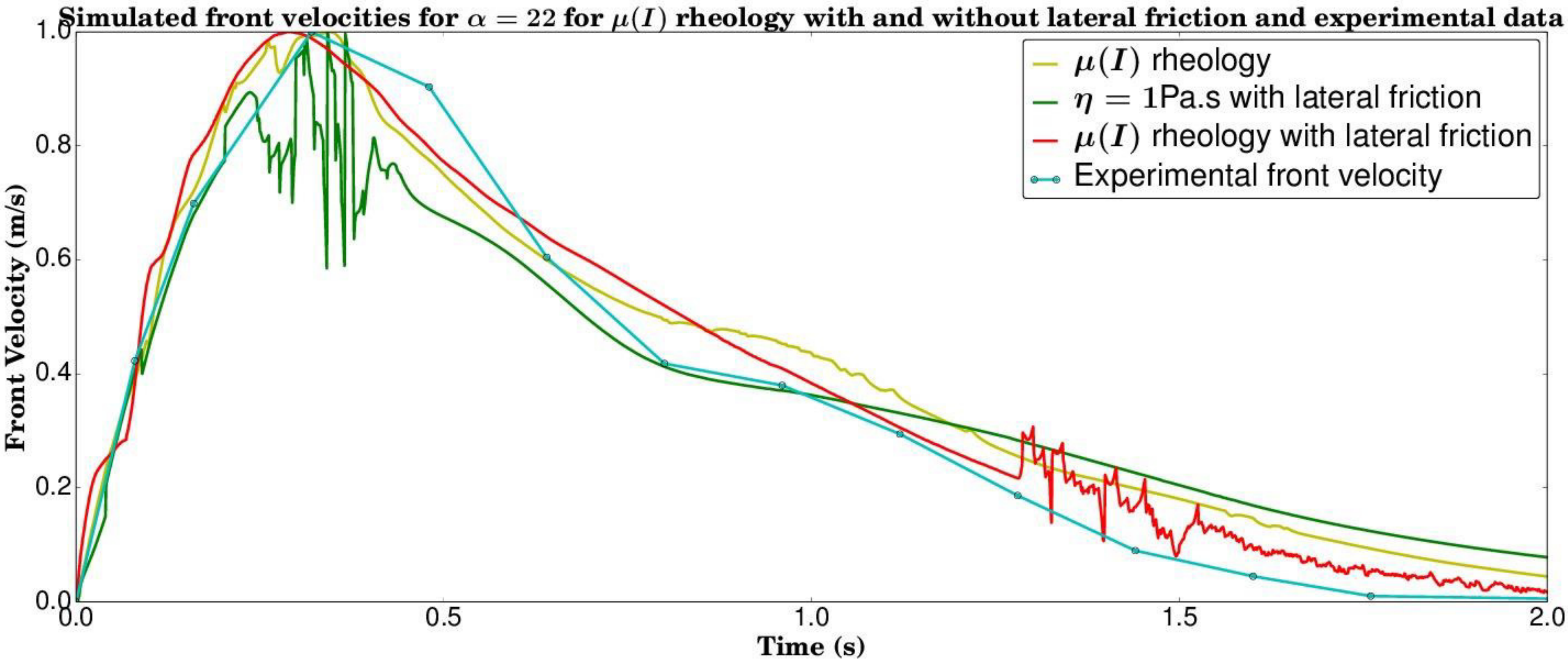
Evolution of front velocities

$\alpha = 19°$



Evolution of front velocities

$\alpha = 22°$



Summary

- The *front propagation* is quite *well reproduced* with both rheologies (and very little differences between them)

- The decreasing of the velocity after the maximum is slower and less sharp in the numerical simulation. The lateral friction term induces a faster decreasing of the velocity in the deceleration phase.

- The **viscoplastic behavior** introduced through $\mu(I)$ rheology is crucial but the spatial variability of the viscosity does not really affect the flow

Perspectives

- The static-flowing transition quickly varies in the *acceleration phase* : the use of an *adaptive time step* could help to provide a better agreement with experimental data at the beginning

- The use automatic mesh refinement could help to better capture the static-flowing transition

- The *dilatancy phenomena* of the material is strong at the beginning (up to 5%) and should be considered in the model (e.g. dilatant Drucker-Prager model)

- The role of the $\mu(I)$ rheology for this highly transient test case remains unclear and should be investigated e.g. on a stationnary uniform flow

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Algorithmique

Constitutive equation

$$\operatorname{trace}(\boldsymbol{D}) = 0, \quad \begin{cases} \boldsymbol{\sigma}' = 2\eta(\|\boldsymbol{D}\|, p)\boldsymbol{D} + \kappa(p)\frac{\boldsymbol{D}}{\|\boldsymbol{D}\|} & \text{if } \boldsymbol{D} \neq 0, \\ \|\boldsymbol{\sigma}'\| \leqslant \kappa(p) & \text{if } \boldsymbol{D} = 0. \end{cases}$$

$$\boldsymbol{D} = \frac{1}{2\eta(\|\boldsymbol{D}\|, p)}\left[1 - \frac{\kappa(p)}{\|\boldsymbol{\sigma}'\|}\right]_+ \boldsymbol{\sigma}',$$

Frictional problem

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0, \quad \begin{cases} \boldsymbol{\sigma}_T = -\mu_C[-\sigma_n]_+ \frac{\boldsymbol{u}_T}{|\boldsymbol{u}_T|} & \text{if } \boldsymbol{u}_T \neq 0, \\ |\ \boldsymbol{\sigma}_T\ | \leqslant \mu_C[-\sigma_n]_+ & \text{if } \boldsymbol{u}_T = 0, \end{cases}$$

$$\boldsymbol{u}_T = -\frac{1}{\eta_f}\left[1 - \frac{\mu_C[-\sigma_n]_+}{|\ \boldsymbol{\sigma}_T\ |}\right]_+ \boldsymbol{\sigma}_T,$$

Same structure $\Rightarrow$ *decomposition-coordination* (augmented Lagrangian) formulation

Algorithmique

Time discretization

$$\rho\left(\frac{\boldsymbol{u}^k - \boldsymbol{u}^{k-1}}{\Delta t} + \boldsymbol{u}^k \cdot \nabla \boldsymbol{u}^k\right) - \mathbf{div}\, \boldsymbol{\sigma}'^k + \nabla p^k = \rho \boldsymbol{f} \quad \text{in} \quad \mathcal{D},$$

$$\operatorname{div}(\boldsymbol{u}^k) = 0 \quad \text{in} \quad \mathcal{D},$$

$$\boldsymbol{D}(\boldsymbol{u}^k) = \frac{1}{2\eta(\|\boldsymbol{D}(\boldsymbol{u}^k)\|, p^k)}\left[1 - \frac{\kappa(p^k)}{\|\boldsymbol{\sigma}'^k\|}\right]_+ \boldsymbol{\sigma}'^k,$$

while the boundary conditions read

$$\boldsymbol{\sigma}^k \boldsymbol{n} = 0 \quad \text{on} \quad \Gamma_s,$$

$$\boldsymbol{u}^k \cdot \boldsymbol{n} = 0, \quad \boldsymbol{u}_T^k = -\frac{1}{\eta_f}\left[1 - \frac{\mu_C[-\sigma_n^k]_+}{|\,\boldsymbol{\sigma}_T^k\,|}\right]_+ \boldsymbol{\sigma}_T^k, \quad \text{on} \quad \Gamma_b.$$

Algorithmique

Step 1. The first step consists in solving the following linear equation of the Stokes type for the velocity field $\boldsymbol{u}^{k,n}$ and the pressure $p^{k,n}$:

$$\operatorname{div}(\boldsymbol{u}^{k,n}) = 0, \tag{31}$$

$$\rho\left(\frac{\boldsymbol{u}^{k,n} - \boldsymbol{u}^{k-1}}{\Delta t} + \boldsymbol{u}^{k,n-1} \cdot \nabla \boldsymbol{u}^{k,n}\right) - \mathbf{div}(r\boldsymbol{D}(\boldsymbol{u}^{k,n})) + \nabla p^{k,n}$$
$$= \mathbf{div}(\boldsymbol{\sigma}'^{k,n-1} - r\dot{\gamma}^{k,n-1}) + \rho \boldsymbol{f}, \tag{32}$$

with the boundary conditions

$$(r\boldsymbol{D}(\boldsymbol{u}^{k,n}) - p^{k,n}\boldsymbol{I} + \boldsymbol{\sigma}'^{k,n-1} - r\dot{\gamma}^{k,n-1})\boldsymbol{n} = 0, \quad \text{on} \quad \Gamma_s,$$

$$\boldsymbol{u}^{k,n} \cdot \boldsymbol{n} = 0, \quad \text{on} \quad \Gamma_b,$$

$$(r\boldsymbol{D}(\boldsymbol{u}^{k,n}) - p^{k,n}\boldsymbol{I} + \boldsymbol{\sigma}'^{k,n-1} - r\dot{\gamma}^{k,n-1})_T$$
$$= -r_f \boldsymbol{u}_T^{k,n} + r_f \delta^{k,n-1} + \boldsymbol{\sigma}_T^{k,n-1}, \quad \text{on} \quad \Gamma_b.$$

Algorithmique

Step 2. First we update the viscosity coefficient $\eta = \eta(\|\boldsymbol{D}(\boldsymbol{u}^{k,n})\|, p^{k,n})$ and the yield limit $\kappa = \kappa(p^{k,n})$. Then, we compute the strain rate multipliers $\dot{\gamma}^{k,n}$ and the slip rate multipliers $\delta^{k,n}$

$$\dot{\gamma}^{k,n} = \frac{1}{2\eta + r}\left[1 - \frac{\kappa}{\|\boldsymbol{\sigma}'^{k,n-1} + r\boldsymbol{D}(\boldsymbol{u}^{k,n})\|}\right]_+ (\boldsymbol{\sigma}'^{k,n-1} + r\boldsymbol{D}(\boldsymbol{u}^{k,n})), \qquad (33)$$

$$\delta^{k,n} = -\frac{1}{\eta_f + r_f}\left[1 - \frac{\mu_C[-\sigma_n^{k,n-1}]_+}{|\,\boldsymbol{\sigma}_T^{k,n-1} - r_f\boldsymbol{u}_T^{k,n}\,|}\right]_+ (\boldsymbol{\sigma}_T^{k,n-1} - r_f\boldsymbol{u}_T^{k,n}), \qquad (34)$$

according to the decomposition–coordination formulation coupled with the augmented Lagrangian method.

Step 3. Finally, we update the stress deviator $\boldsymbol{\sigma}'^{k,n}$ and the tangential stress $\boldsymbol{\sigma}_T^{k,n}$ using

$$\boldsymbol{\sigma}'^{k,n} = \boldsymbol{\sigma}'^{k,n-1} + r(\boldsymbol{D}(\boldsymbol{u}^{k,n}) - \dot{\gamma}^{k,n}),$$

$$\boldsymbol{\sigma}_T^{k,n} = \boldsymbol{\sigma}_T^{k,n-1} - r_f(\boldsymbol{u}_T^{k,n} - \delta^{k,n}).$$