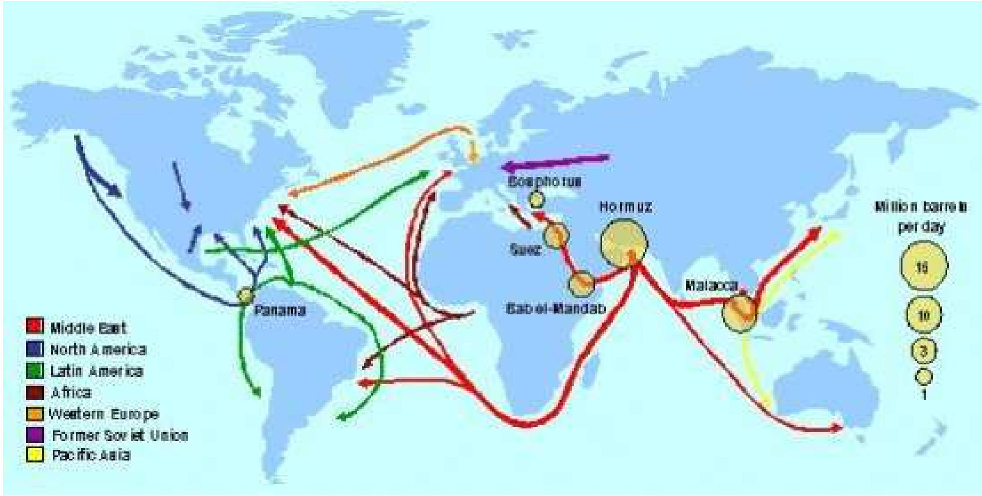


Littoral erosion & extreme events

Hazard quantification in oil transport on seas

Quantification of buried oil in the intertidial beach zone

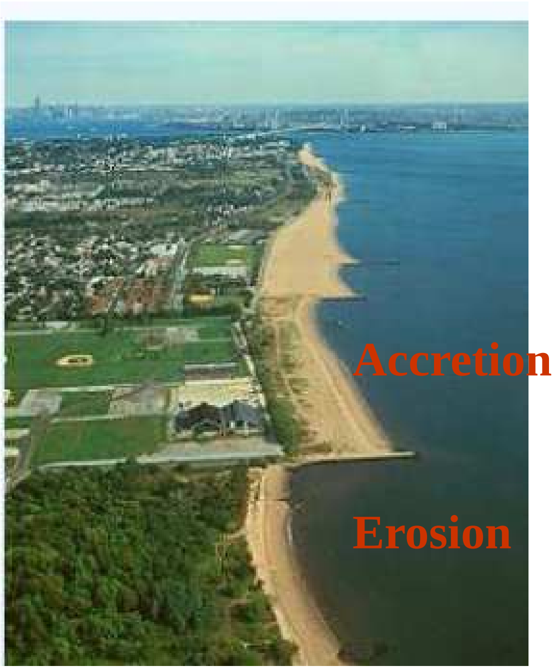
Summary

Morphodynamics by minimization principle:
fluid model + bottom motion minimizing a given energy

+ Uncertainties on bed characteristics

+ Extreme scenarios

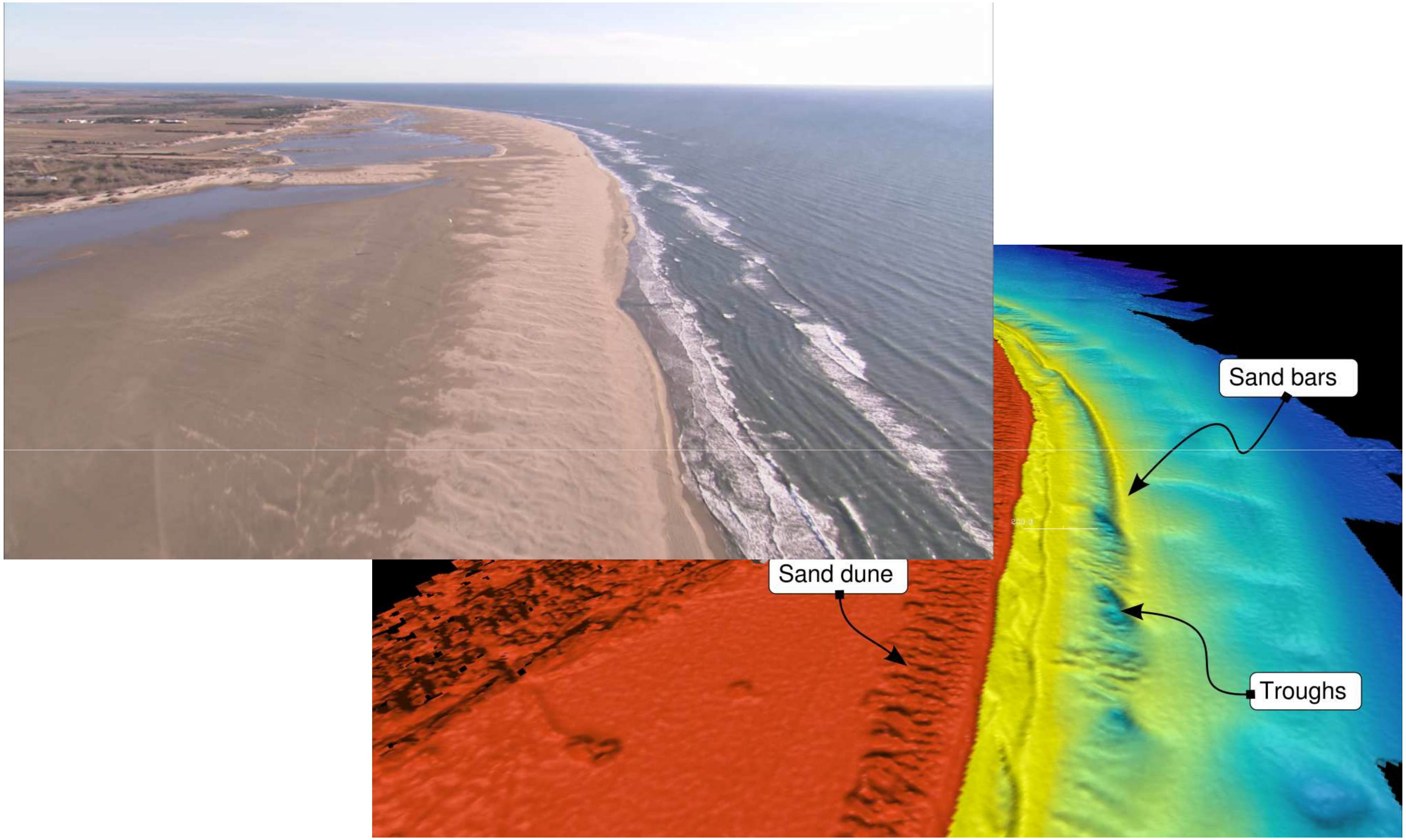
Erosion and storms



Erosion concerns 70% sandy beaches worldwide

Sète

Coupling shallow water & soft bottom



Think of other multi-physics coupling through interactions at boundaries

Fluid/Bottom model

Dependency chain: $\psi \rightarrow \{\mathbf{U}(\psi,\tau), \tau \in [0,T]\} \rightarrow J(\psi,T)$

Flow state equation: $\mathbf{U}_t + F(\mathbf{U},\psi) = 0, \quad \mathbf{U}(0) = \mathbf{U}_0(\psi)$

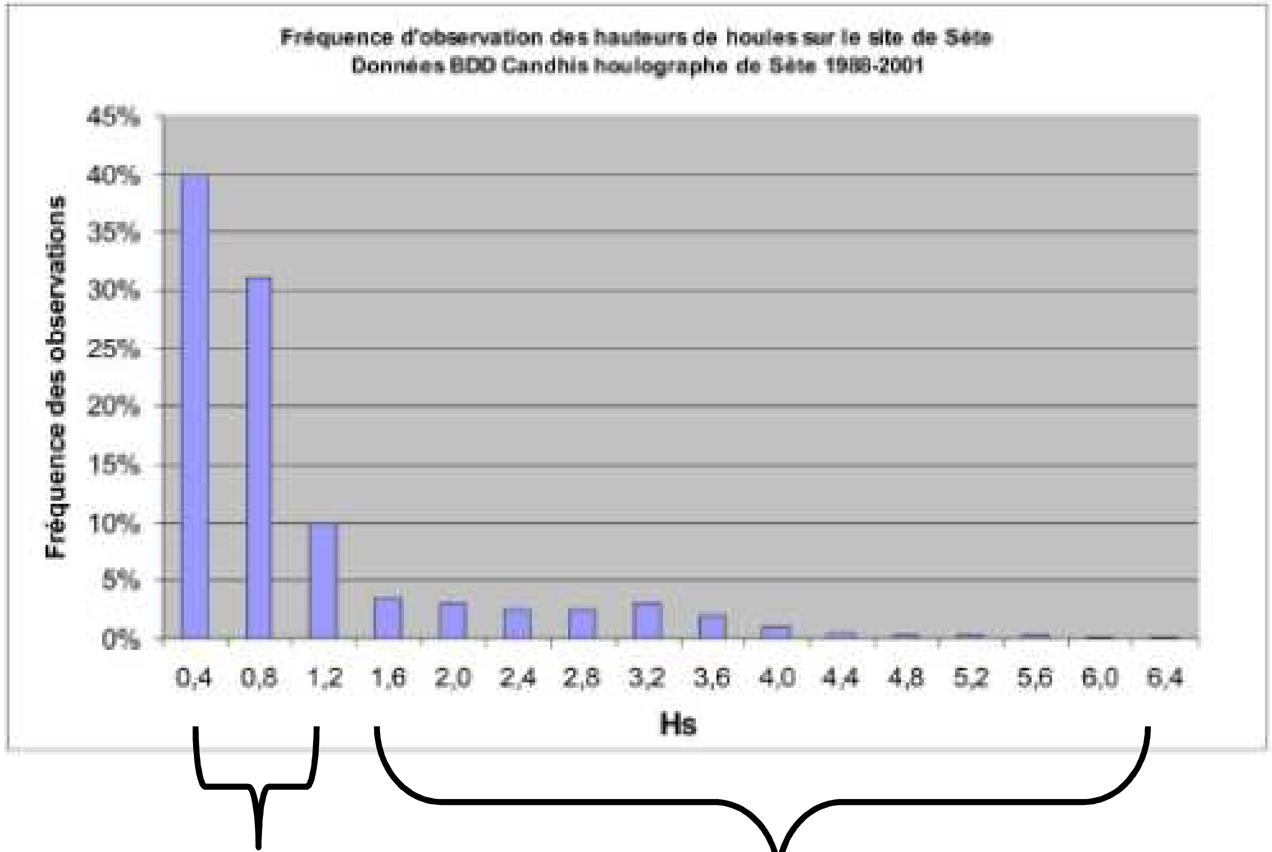
Cost fct: $J(\psi,T) = \int_{(0,T)} j(\psi, \mathbf{U}(\psi,t))$

Bed motion: $\partial_t \psi = -\rho(t,x)\nabla_\psi J, \qquad \psi(t=0,x) = \psi_0(x) = \text{given},$

Bed time and space variability through its response to flow perturbations.
Aleatoric uncertainties also present in initial and boundary conditions.
Epistemic uncertainties due to model & numerics.

→ Same platform used to design beach protection devices (geotube, sand dune, groyne, etc).

Erosion and waves



Constructive waves

Destructive waves

Example of functional

for beach morphodynamics simulations

T: Time interval of influence

Ω *:* observation domain

$$J(U(\psi)) = \int_{t-T}^{t} \int_{\Omega} (\frac{1}{2} \rho_w g \eta^2 + \rho_s g (\psi(\tau) - \psi(t-T))^2) d\tau d\Omega$$

$$\eta(\tau, x, \psi) = h(\tau, x, \psi) - \frac{1}{T} \int_{t-T}^{t} h(\tau, x, \psi) d\tau$$

Hypothesis:

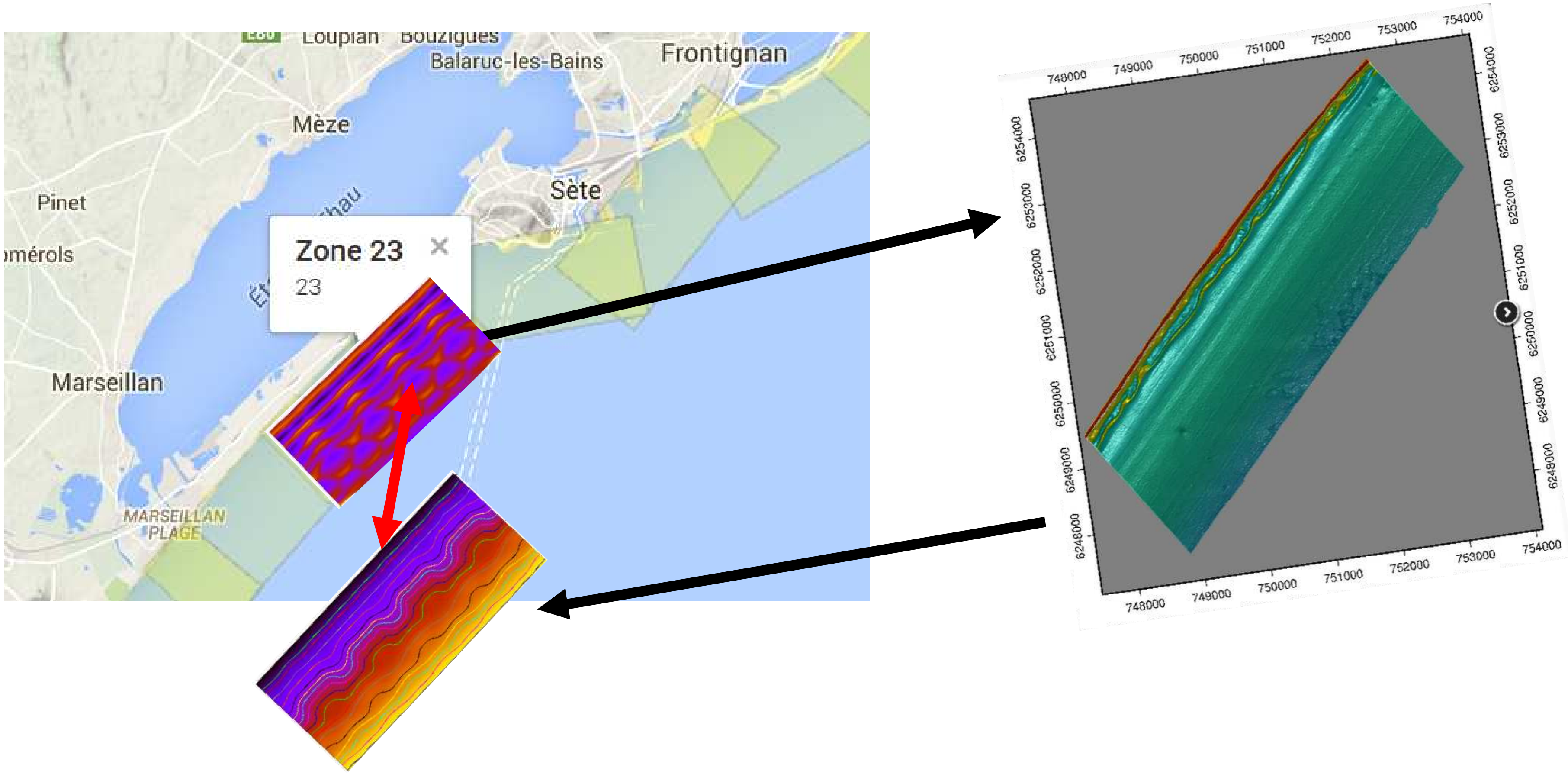
The bed adapts in order to reduce water kinetic energy with ‘minimal’ sand transport

Coupling shallow water & bottom

http://www.soltc.org/database/lidar

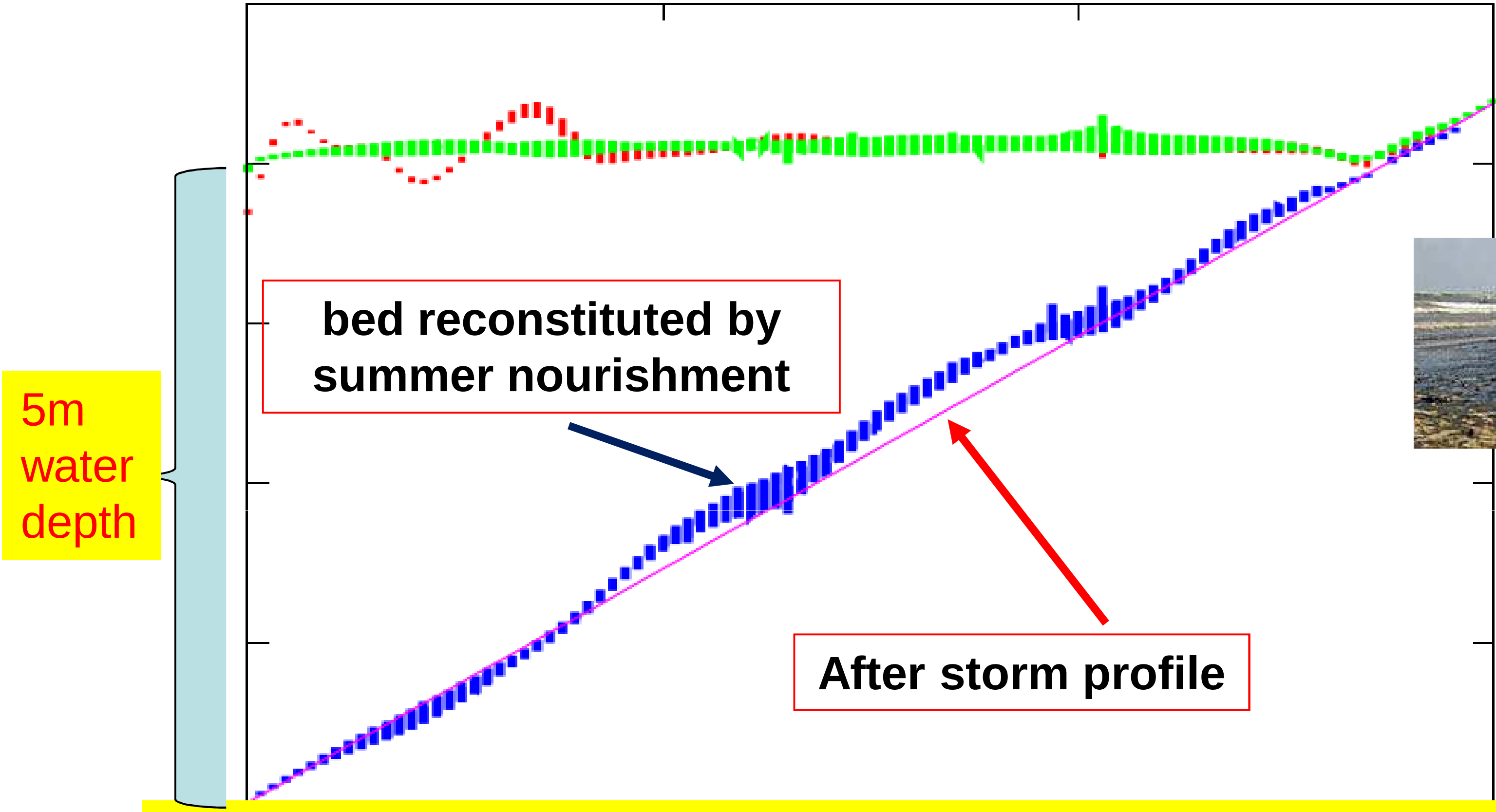
Coupling shallow water & bottom

http://www.soltc.org/database/lidar



OildeBeach project

How deep oil is buried ?

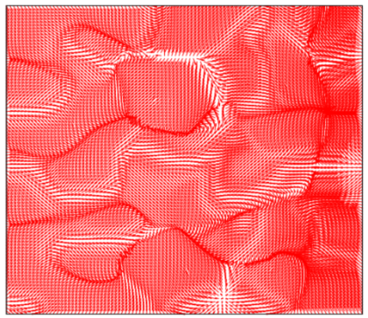


Cross-shore distance: 150m

Oil might be covered by 40cm of sand in some area, corresponding to on site observations (Prestige oil spill)

It can reappear next winter !

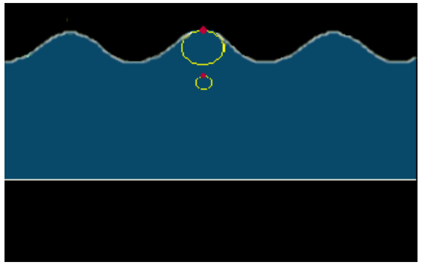
Pertinent with strong tidal coefficient (Saint-Malo 28-116) (Piriac 40-100)



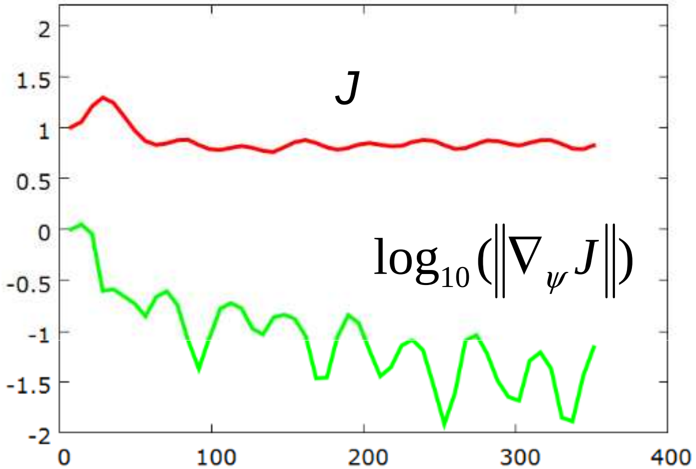
Equivalent bottom velocity

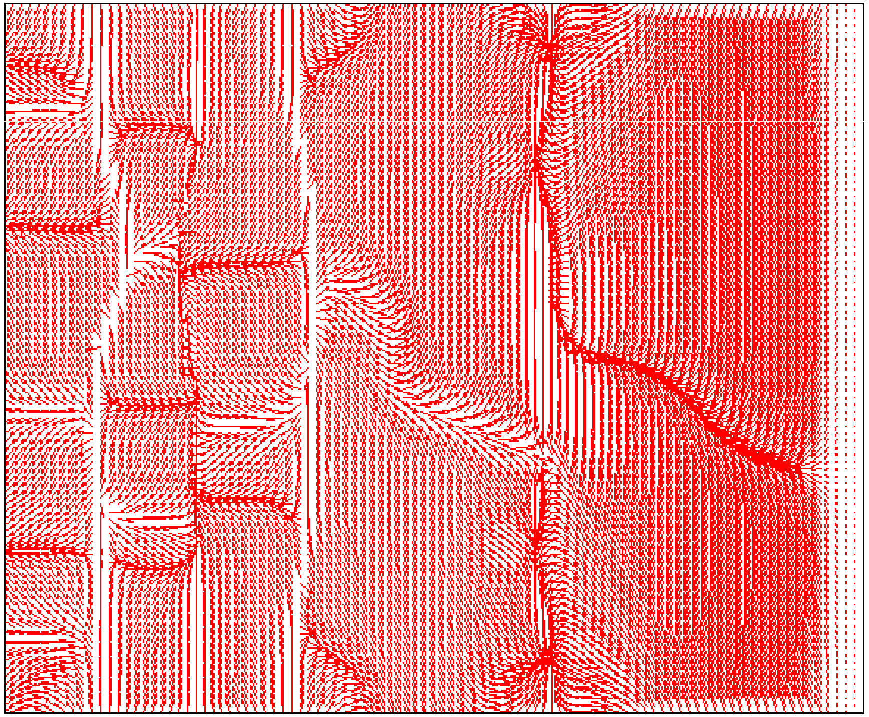
$$\psi_{,t} = -\rho \nabla_{\psi} J = -V \nabla_{x} \psi$$

$$\left|\psi_{,t}\right| \leq \left|U_{orb}(z=-h)\right| \left\|\nabla_{x} \psi\right\|$$



Web Y. Place





Impact of bed characteristics uncertainty

Extreme scenarios

Knowing the PDF of the bed characteristics
and
given a confidence level, provide extreme evolution scenarios for the bed

Quantile-based extreme scenrios

Knowing the PDF of the uncertainties on the bed receptivity, define two extreme scenarios for the bed receptivity and therefore for our fluid-structure coupling.

$$0 \le \rho(x) = \rho_0(x) + \mathrm{VaR}^{\alpha}_{\pm}(x), \text{ with } \mathrm{VaR}^{\alpha}_{-} \le 0 \le \mathrm{VaR}^{\alpha}_{+}$$

Knowing the PDF of the uncertainties of a control parameter x, define two quantile-based extreme scenarios for the variations:

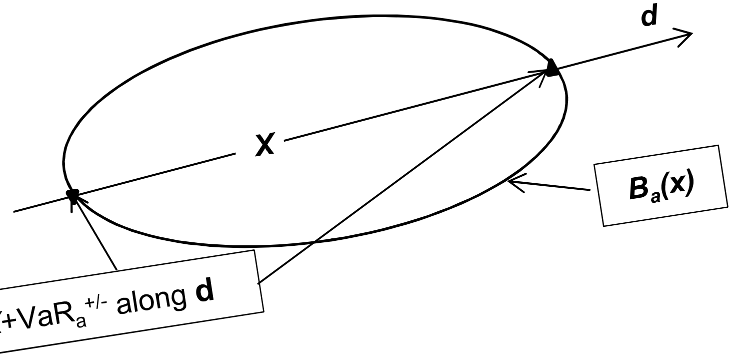
$$X = x + \mathrm{VaR}^{\alpha}_{\pm}(x), \text{ with } \mathrm{VaR}^{\alpha}_{-} \le 0 \le \mathrm{VaR}^{\alpha}_{+} \qquad \text{(defined component by component)}$$

If Gaussian PDF:

$\mathrm{VaR}^{0.99} = 2.33$ and $\mathrm{VaR}^{0.95} = 1.65$ for $N(0,1)$

and $\mathrm{VaR}^{\alpha}(N(0,\sigma(x))) = \sigma(x)\ \mathrm{VaR}^{\alpha}(N(0,1))$

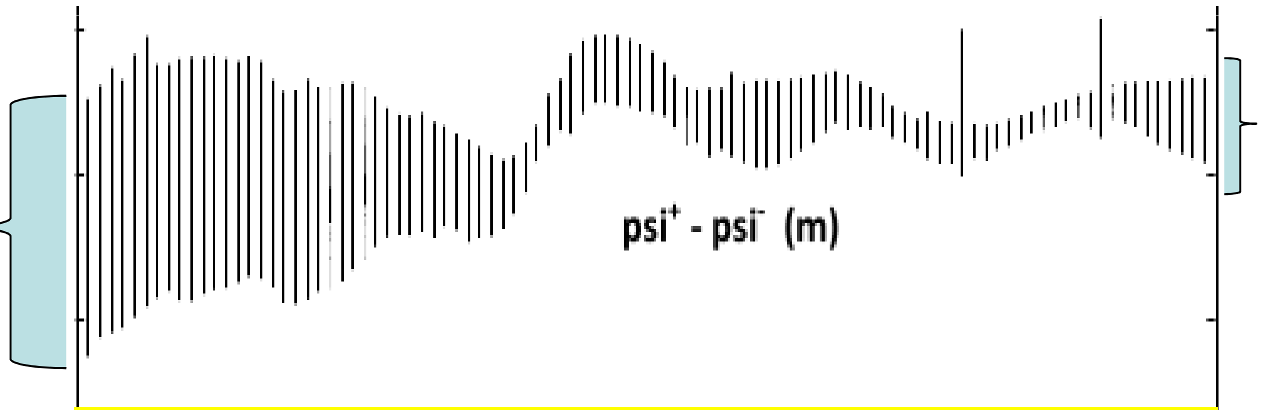
and $\mathrm{VaR}^{\alpha}_{-} = -\mathrm{VaR}^{\alpha}_{+}$



Bed variability…….due to………bed mobility variability

Assumption: sand mobility variability increases toward the beach: from 0 to 50%
This also accounts for imperfect modelling (epistemic UQ)

~10cm Variability for 5m water depth: 2%



~5cm Variability for 20cm water depth: 25%

Cross-shore distance: 150m

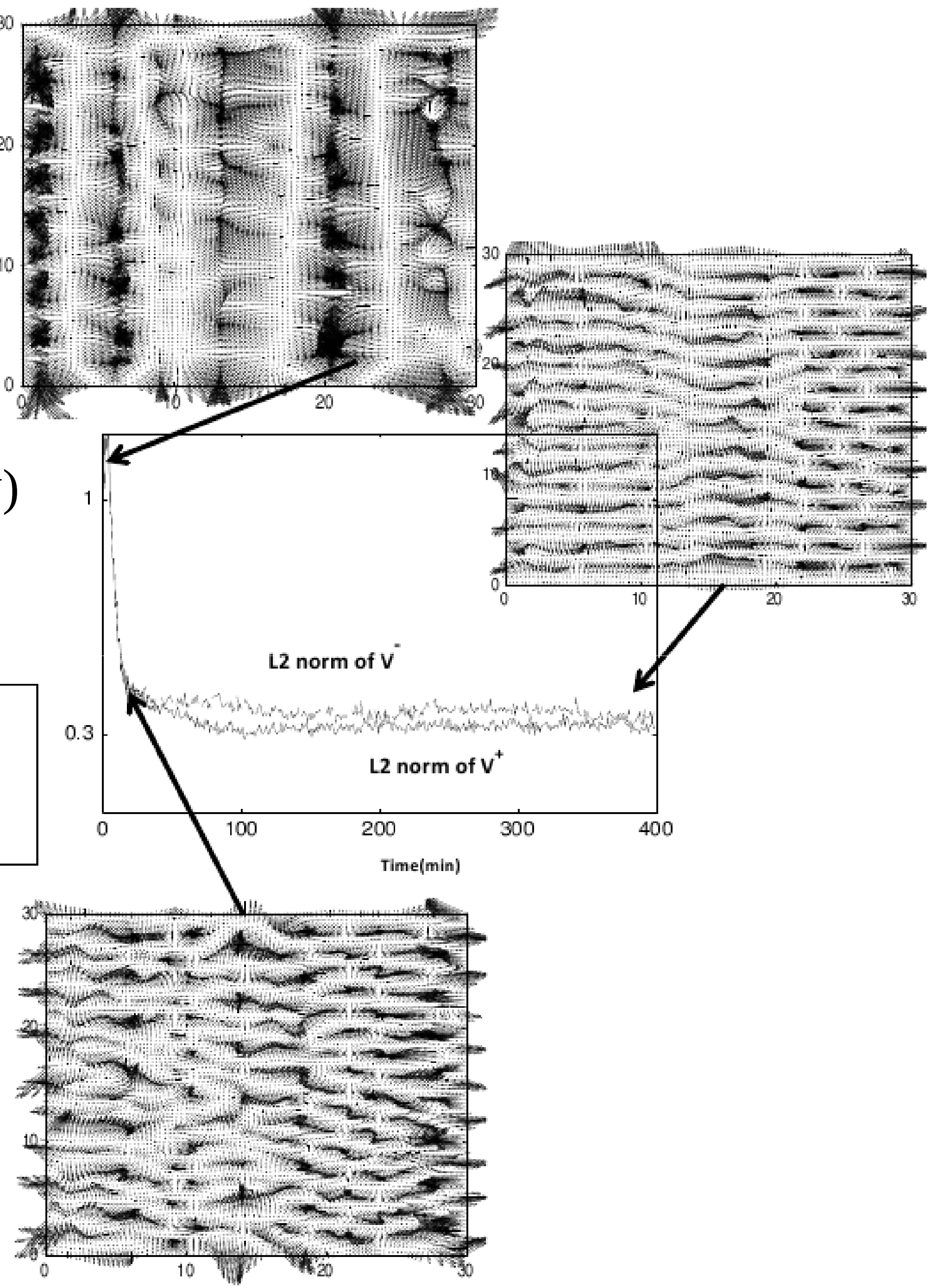
Bottom equivalent velocity V

$$\psi_t = -\rho\nabla_\psi J = -V\,\nabla_x\psi$$

Formally, $V = (\rho\nabla_\psi J / \partial_{x1}\psi, \rho\nabla_\psi J / \partial_{x2}\psi)$

Remark : Conservation, $\int_\Omega \psi_t = 0$

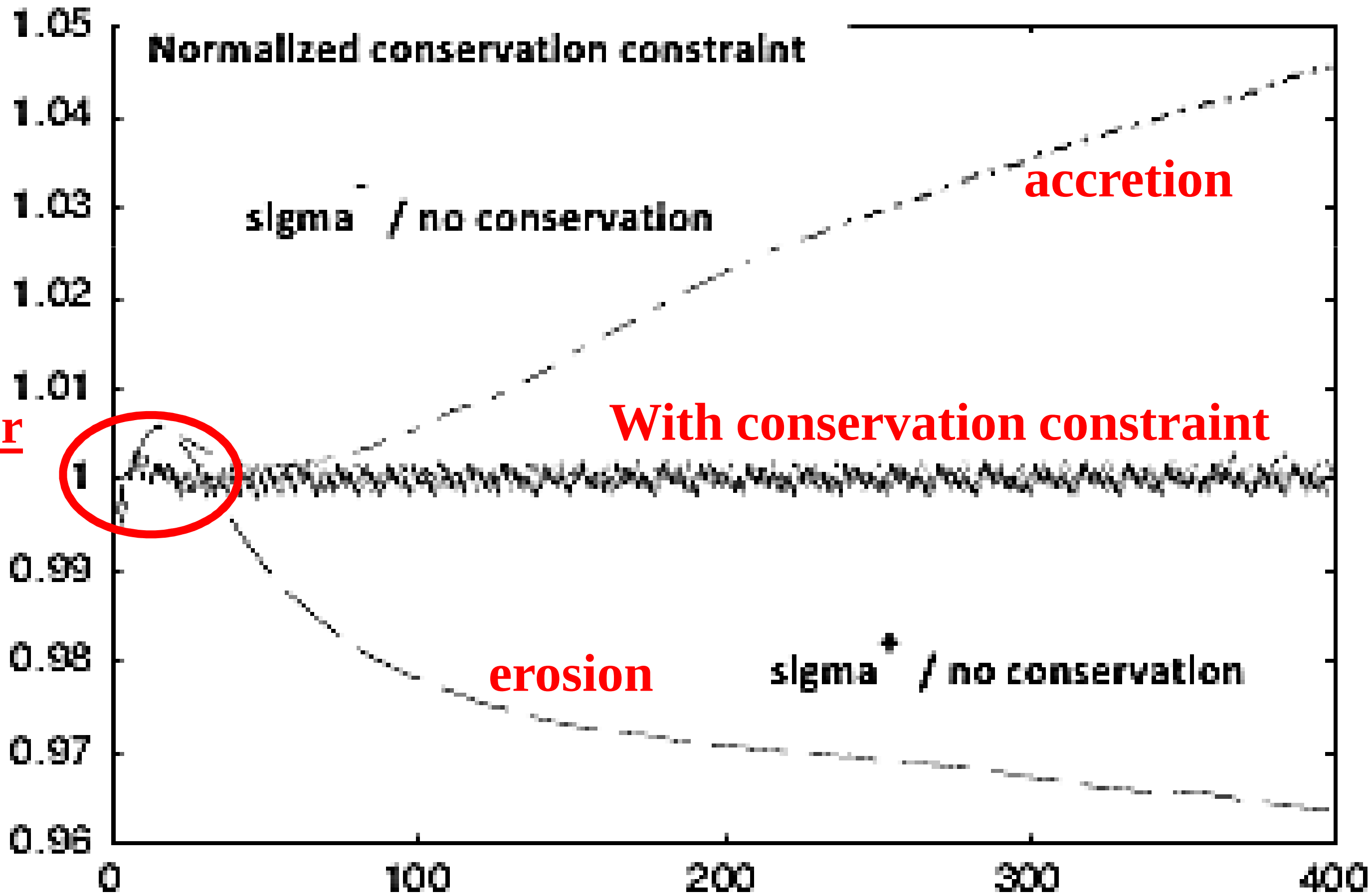
=> Optimization under constraint:
Experiences in basin cannot represent open sea.



Differences between basin and open sea

Sogreah basin

Impact of conservation constraint + sand variability



Package

Impact of state uncertainty

Adjoint-based & extreme scenarios construction of bed covariance matrix

Suppose $\mathbf{x}$ zero-mean, $u = L\mathbf{x}$, and $u \leftarrow u - \mu$

$$Cov_u = \mathbb{E}(uu^t) = \mathbb{E}(L\ \mathbf{x}\mathbf{x}^t\ L^t) = L\ \mathbb{E}(\mathbf{x}\mathbf{x}^t)\ L^t = L\ Cov_{\mathbf{x}}\ L^t$$

If nonlinear, $L = \mathcal{J} = \partial u / \partial \mathbf{x}$: $Cov_u = \mathcal{J}\ Cov_{\mathbf{x}}\ \mathcal{J}^t$

$$Cov_{\mathbf{x}} = \mathcal{J}^{-1}\ Cov_u\ \mathcal{J}^{-t} = \left(\mathcal{J}^t\ Cov_u^{-1}\ \mathcal{J}\right)^{-1}$$

Suppose $\nabla_{\mathbf{x}} j$ available by adjoint.

$$\nabla_{\mathbf{x}} j = \frac{\partial j}{\partial \mathbf{x}} + \left((\frac{\partial j}{\partial u})^t\ \frac{\partial u}{\partial \mathbf{x}} \right)^t = \frac{\partial j}{\partial \mathbf{x}} + \left((\frac{\partial j}{\partial u})^t \mathcal{J} \right)^t$$

Therefore,

$$\mathcal{J} = (\frac{\partial j}{\partial u})^{-t}\ (\nabla_{\mathbf{x}} j - \frac{\partial j}{\partial \mathbf{x}})^t$$

In least-square sense:

$$\mathcal{J} = \left((\frac{\partial j}{\partial u})(\frac{\partial j}{\partial u})^t \right)^{-1}\ \frac{\partial j}{\partial u}\ (\nabla_{\mathbf{x}} j - \frac{\partial j}{\partial \mathbf{x}})^t$$

To be compared to the DES based covariance:

$$Cov_{\mathbf{x}^{\pm}} = \mathbb{E}((X^{\pm})(X^{\pm})^t) - \mathbb{E}(X^{\pm})\mathbb{E}(X^{\pm})^t \sim (X^{\pm})(X^{\pm})^t - (\overline{X^{\pm}})(\overline{X^{\pm}})^t$$

with $\overline{X^{\pm}} = ((\mathbf{x}^*)^+ + (\mathbf{x}^*)^- - 2\mathbf{x}^*)/2$

Two-step application

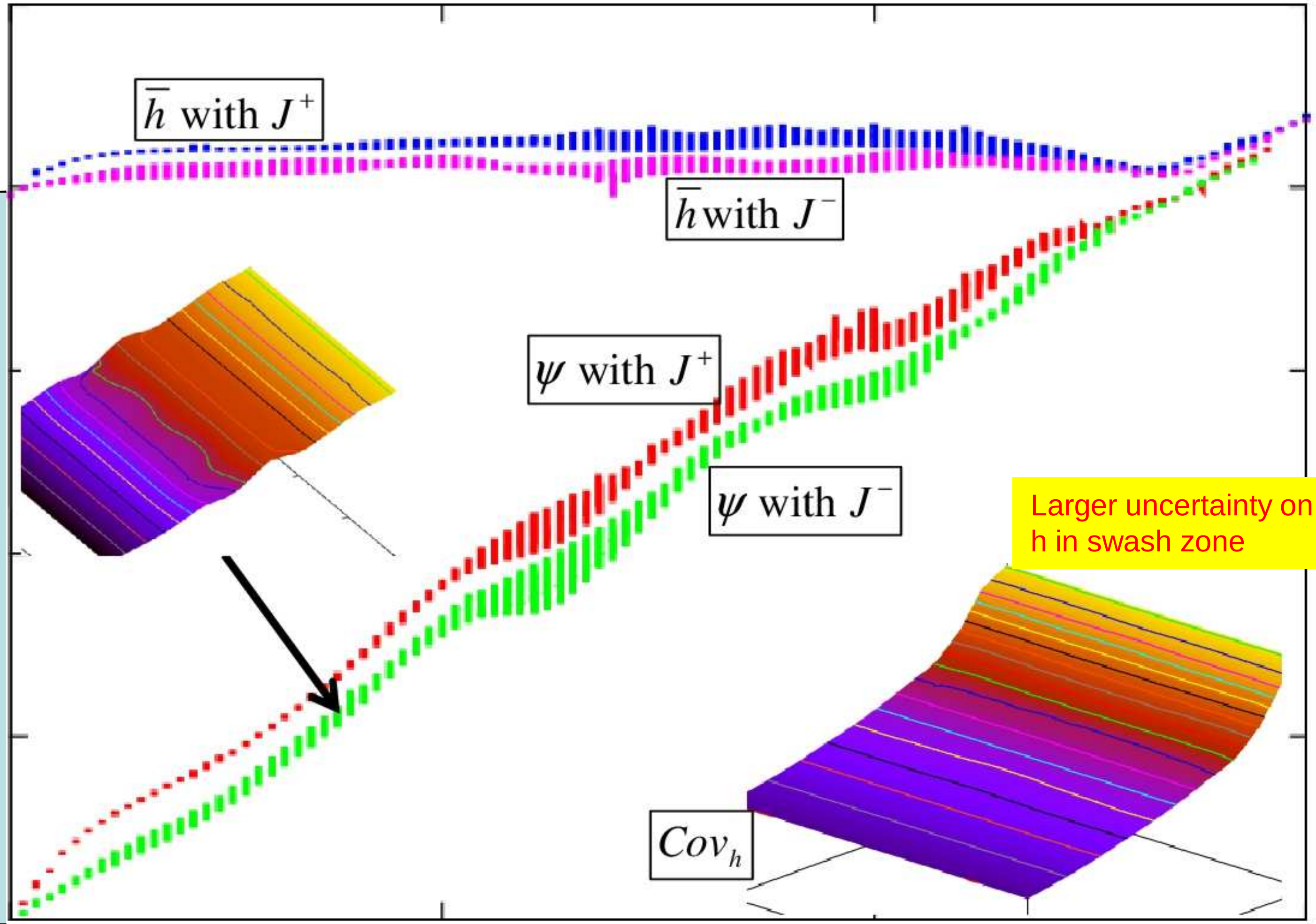
1/ $x^* = \arg\min_x (j(u(x)))$

2/ Cov_x with $j \leftarrow j(u(x)) + \|u(x) - u(x^*)\|$

Cov_{u^*} known & modelling

aleatoric and epistemic uncertainties

Extreme scenarios due to state variability (epistemic or aleatoric)



~5m water depth

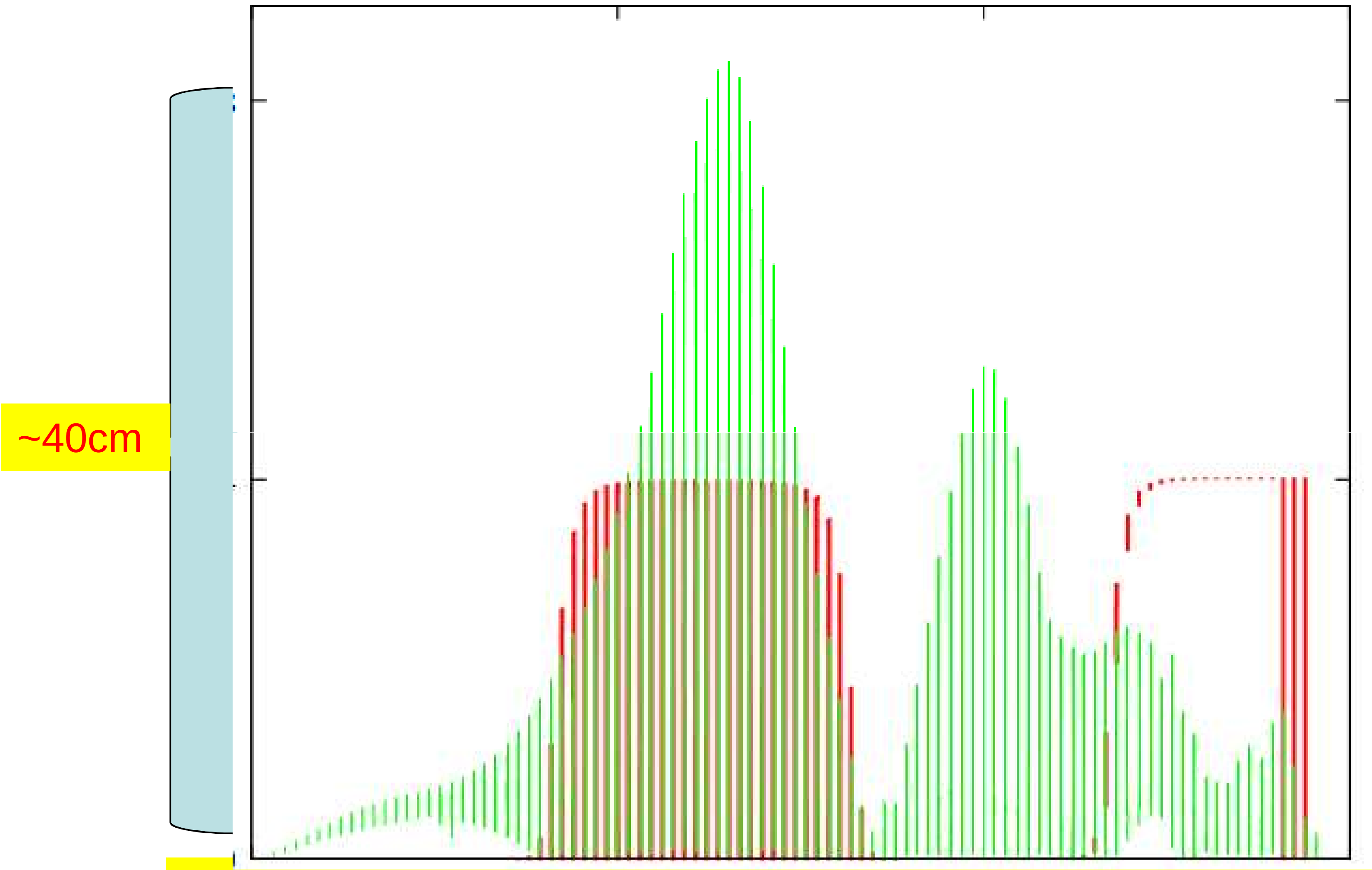
Larger uncertainty on h in swash zone

Cross-shore distance: 150m

5% uncertainty on water level induce an uncertainty on the sand thickness of 40cm !

Bed covariance

Extreme scenarios vs. Adjoint-based



Cross-shore distance: 150m

State uncertainty induces an uncertainty on the sand thickness of 40cm !

Remarks

- Morphodynamics by minimization principle.

-VaR-based extreme scenarios.

-Bed covariance (extreme scenarios & adjoint-based) accounting for epistemic & aleatoric state variability

- Need to account for bed variability during the coupling and not only eventually through engineering margins.

- Open sea is not basin.

- Useful to estimate how deep oil might be present after summer bed beach reconstruction

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(Aeronautics, Littoral, Seismic/reservoir, Others)

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