A three-layers depth-averaged model for sediment transport considering mass exchange

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Abstract

In this work we deduce a mathematical model for the sediment transport problems. Most of the models in the literature concerning this issue hold some of the different kind of transport occurred in rivers: bed-load, suspended-load or wash-load. Additionally, some of them treat the task of the mass exchange in this system what becomes one of the more important effect to be considered.

The objective here is to propose a complete model that accounts for bed load, suspended load and wash load transport at the same time, together with the capability of reproducing erosion and deposition effects. For this aim we use several mathematical tools in order to take into account the physical environment of the problem: depth-average, asymptotic approximation, different time scales for hydrodynamic and morphodynamic components, mass and momentum conservation.
We consider the fluid-solid mixture model proposed by Jackson [4] as a basis and we follow different derivations to obtain the dynamics for the three layers. The top and middle layer are defined respectively as a wash and suspension layer with different particle concentrations, so we derive a two-phase model based on [2] considering same velocity for both phases. For the layer below, we follow the derivation developed in [1] to obtain an Exner type model for arbitrary sloping beds.
Erosion, suspension and deposition are considered in the whole system through appropriate boundary conditions based on mass and momentum conservation at the interfaces. The erosion/deposition rates are defined following [5, 3]. Finally, several numerical tests will be presented.

References

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