A Newton method for viscoplastic flows

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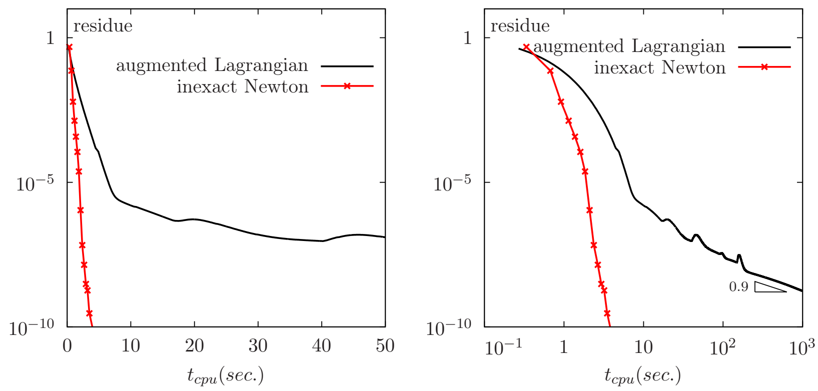

Abstract – A damped Newton algorithm for viscoelastic fluid flows is presented [1]. This algorithm bases on a projection formulation of the viscoplastic problem and an efficient preconditioned iterative solver for the singular Jacobian. A demonstration is provided by computing a viscoplastic flow in a pipe with a square cross section and performance are compared with the augmented Lagrangian algorithm.

The numerical resolution of viscoplastic fluid flows is still a challenging task [2]. Two main main approaches are considered: the augmented Lagrangian algorithm [3], which is accurate and slow, and the regularization approach, which is faster but could be less accurate, as it solves a modified problem. This work is a contribution to an ongoing effort for the development of faster algorithms than the augmented Lagrangian for the resolution of the unregularized viscoplastic model. One of the most efficient algorithm to solve nonlinear problems is the Newton method, due to its superlinear convergence properties. Applying the Newton method to the unregularized viscoplastic problem leads to a singular Jacobian matrix. In this work, we address directly the singularity of the Jacobian matrix in the Newton method in order to preserve the superlinear convergence. We present a Newton method for the unregularized viscoplastic fluid flow problem [1]. It leads to a quadratic convergence for Herschel-Bulkley fluids when $0 < n < 1$, where n is the power law index. Performances are enhanced by using the inexact variant of the Newton method and, for solving the Jacobian system, by using an efficient preconditioner based on the regularized problem. A demonstration is provided by computing a viscoplastic flow in a pipe with a square cross section. Comparisons with the augmented Lagrangian algorithm show a dramatic reduction of the required computing time while this new algorithm provides an equivalent accuracy for the prediction of the yield surfaces. The figure plots a comparison of the present inexact preconditioned damped Newton algorithm with the classical Uzawa/augmented Lagrangian method. The augmentation parameter r for the augmented Lagrangian algorithm (AL) has been optimized for the present mesh (a pipe section with $h = 1/80$ and 5781 elements) in order to present the best possible convergence rate. Both algorithms are implemented in the `Rheolef` free software FEM library [**?**]. Observe the dramatic efficiency of the Newton algorithm, which converges in less than 5 seconds to a residue less than 10^{-10} while the AL becomes slower and slower in semi-log scale and adopts an asymptotic behavior, as shown in semi-log scale, where the residue behaves as $1/t^{\alpha}$, with $\alpha \approx 0.75$. After about 15 minutes, the residue is of about 10^{-7} and, by extrapolation, reaching 10^{-10} would requires one day of computation.

References

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