HYPERBOLIC MODELS OF BUBBLY FLUIDS: VARIATIONAL APPROACH

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Physical processes occurring in the continuous media are often described by mathematical models which admit a variational formulation. One of the examples is the dispersive bubbly fluid model. Although dispersive models capture the physics better, some complexities in imposing the boundary conditions are common in the numerical resolution of such systems. One of the ways to avoid these difficulties consists in construction of an approximate hyperbolic model which takes form of the original dispersive model only in some limit. The new model should be unconditionally hyperbolic to make sure that the corresponding Cauchy problem is well-posed. This approach was already implemented in [1] for Serre-Green-Naghdi (SGN) model which describes the dispersive water waves. One has to mention also [2], where a hyperbolic conservative extension of the governing equations was obtained without the variational approach. Like SGN model, the original bubbly fluid equations are Euler-Lagrange equations for some Lagrangian. To construct the new model, the original Lagrangian is replaced by the "augmented" Lagrangian which tends to the original one in some limit.

The new approximate unconditionally hyperbolic model is derived. The Riemann problem is, in particular, numerically solved.

REFERENCES

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