Consistent models for open-channel flows in the smooth turbulent regime

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Abstract

Departing from the free surface Navier-Stokes equations with a mixing-length model for the law of viscous stress to represent the eects of turbulence, complemented by the relevant boundary conditions, in a two-dimensional setting, we derive a family of consistent reduced approximate models under the assumption of shallow water. To do so, we perform a rst order asymptotic expansion of the wall shear stress and the uid velocity eld in the long wave regime with the help of an iterative scheme. We obtain a consistent description of the turbulent boundary layer and recover the well-known empirical frictions laws of Chezy and Colebrook-White. We next obtain the Saint-Venant model at the leading order and a three-equation model at the next order of approximation, these family being consistent approximations of the Navier-Stokes model. Finally we investigate stability of our models, existence of roll waves, and compare them to some of the few existing in the literature.