Ecoulements de fluides viscoplastiques : expériences et simulations

Débriefing de l'un des projets Tellus INSU - INSMI 2016

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5e Ecole du GdR CNRS EGRIN
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Le projet

L'équipe :

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Guillaume Chambon : IRSTEA Grenoble
Li-Hua Luu (Post-Doc), Pierre Philippe : IRSTEA Aix

Objectifs :

Comparer des simulations et des expériences physiques à base de rhéologie Bingham (ou HB)

... en particulier les zones de transitions fluides / solides

... en particulier dans des configurations 3D à surface libre

Débriefing du travail réalisé en 2016

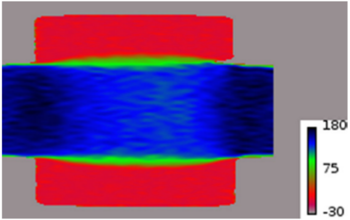
Deux parties :

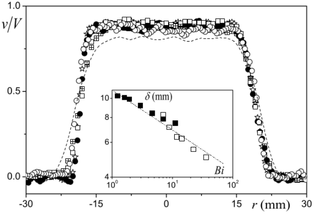
- écoulements confinés en cavité
- schémas numériques **W-B 2D** pour Saint-Venant Bingham

Outline

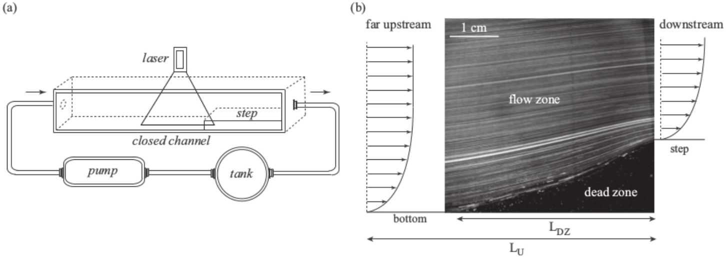
Ecoulements en cavité : 2 cadres expérimentaux

♣ Chevalier et al. EPL 2013 : mesures IRM





♣ Luu et al. PRE 2015 : étude de la "marche" par PIV



Models

Rk. Previous experiments : fluids are Herschel-Bulkley

However, we simplify to Bingham constitutive law :

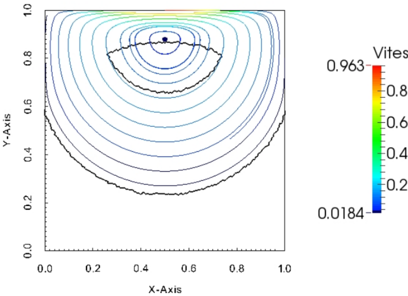
$$\begin{cases} \tau = 2D(\mathbf{u}) + B\dfrac{D(\mathbf{u})}{|D(\mathbf{u})|} \Leftrightarrow D(\mathbf{u}) \neq 0 \\ |\tau| \leqslant B \Leftrightarrow D(\mathbf{u}) = 0. \end{cases} \tag{1}$$

$$\begin{cases} -\nabla.\tau + \nabla p = 0 \\ \nabla.\mathbf{u} = 0, \end{cases} \tag{2}$$

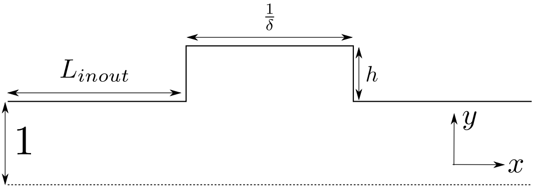
Interestingly, allows to already retrieve non trivial behaviors.

Reminder : lid driven cavity : $x \in \mathbb{R}^2, \mathbf{u} \in \mathbb{R}^2$

- famous benchmark, Newtonian or not
- dead zones : bottom
- plug : top "almond"
- color lines : streamlines



The code



with L_{inout} large enough.

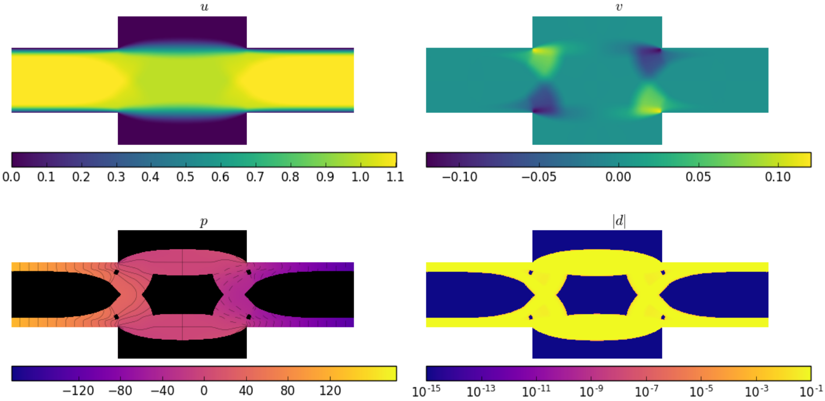
Boundary conditions :

- (cartesian) axial symmetry w.r.t. $x-$axis,
- On the walls : $\mathbf{u} = 0$,
- Inlet/Outlet : $\mathbf{u} = (u_{pois}, 0)$. ∇p s.t. $\int_0^1 u(0, y)\mathrm{d}y = 1$.

Under the hood : (in short)

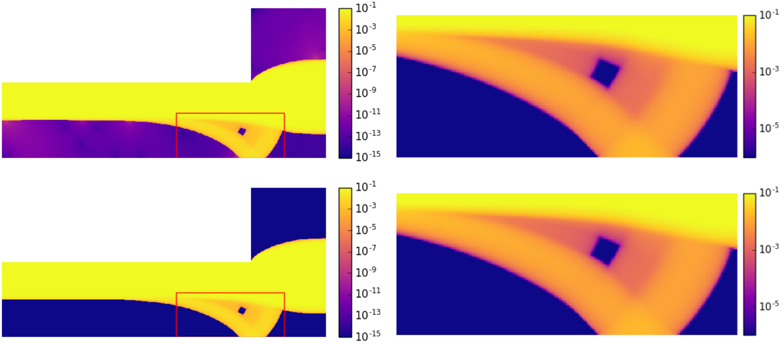
- structured MAC grids, Finite Diff & Augmented Lagrangian
- MUMPS - MPI - F90 parallelization for linear systems

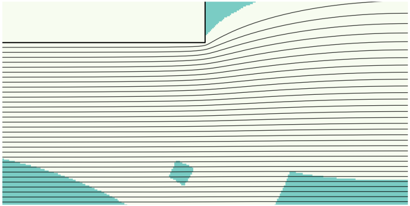
Typical velocity, pressure and $|d|$ ($\leftarrow D(\mathbf{u})$)



$\delta = 0.25$, $h = 1$ and $B = 20$.

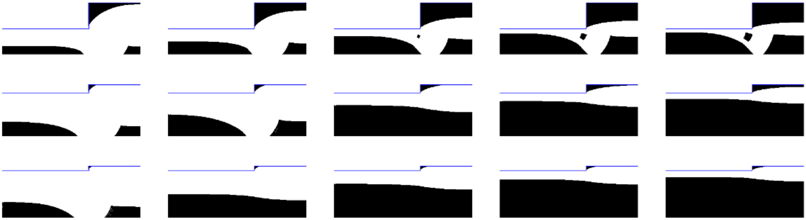
Plug and pseudo-plug zones with streamlines





Top : Zoom on Pseudo-plug := Putz et al. (2009)
Left : Streamlines and plug zones (green)

Different shapes of plug zones

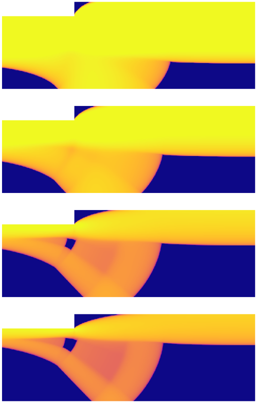


From left to right, $B = 2$, 5, 20, 50 and 100.
Good adequation with the results of Roustaei et al.[2]

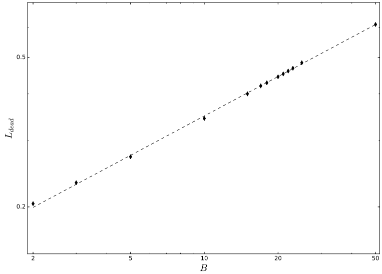
2. A. Roustaei, A. Gosselin, and I. A. Frigaard : JNNFM **220** :87-98 - 2015

Horizontal dead zone length for long cavities

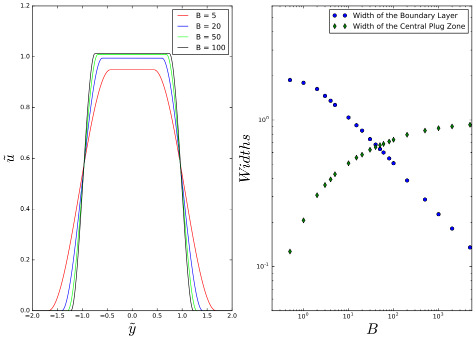
$h = 0.2$ and $\delta = 0.2$.
L_{dead} : horizontal length of the patches of D.Z. in the corner.



Left : $|D(\mathbf{u})|$ for $B = 2 \to 50$
Below : L_{dead} as a function of B with linear fit (slope 0.346).

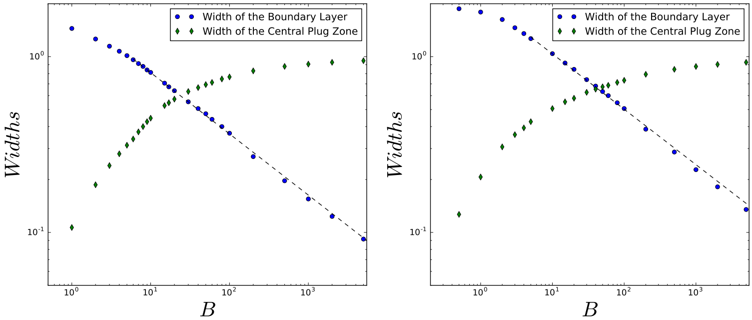


The law of the boundary layer - numer. resul. (1)



Left : Superposition of velocities in the middle of the cavity for different B with $\delta = 0.25$ and $h = 1$.
Right : Boundary layer's width as a function of B

The law of the boundary layer - numer. resul. (2)

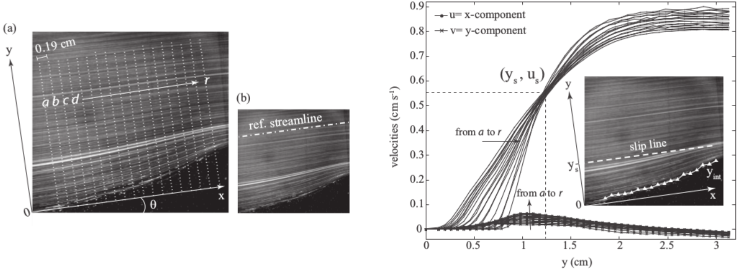


Two different cavity lengths : $\delta = 0.5$ (left) and 0.25 (right).
Linear fits show a slope of respectively -0.348 and -0.315.

Rk 1 : slope -0.2 for Chevalier et al.

Rk 2 : slope -0.33 for the "Oldroyd's 1947" scaling

Luu et al : experimental evidence of a slip line



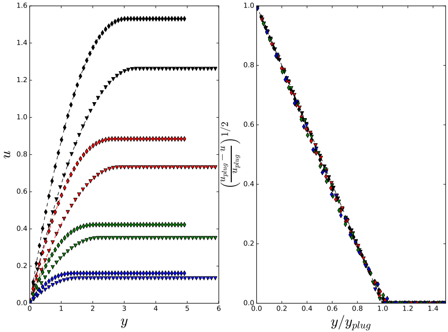
Key observation : by tilting the frame by a certain angle θ, one can observe that the velocity profiles seem to intersect in the same point (y_s, u_s).

The line $y = y_s$ is called a *slip line*

Far up or downstream in our configuration

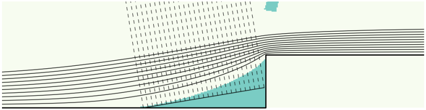
The Poiseuille flow should satisfy the equation :

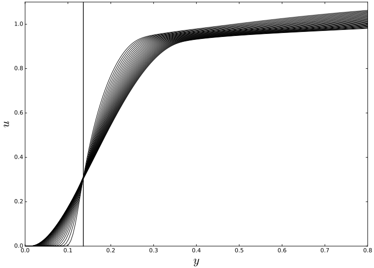
$$\left(\frac{u_{plug} - u_{pois}(y)}{u_{plug}}\right)^{1/2} = \begin{cases} 1 - \dfrac{y}{y_{plug}} \text{ if } 0 \leqslant y \leqslant y_{plug} \\ 0 \text{ if } y > y_{plug}. \end{cases} \quad (3)$$



Velocity profiles far up and downstream for $\delta = 1/12$, $h = 0.2$ and different B between 3 and 25.

Numerical reproduction

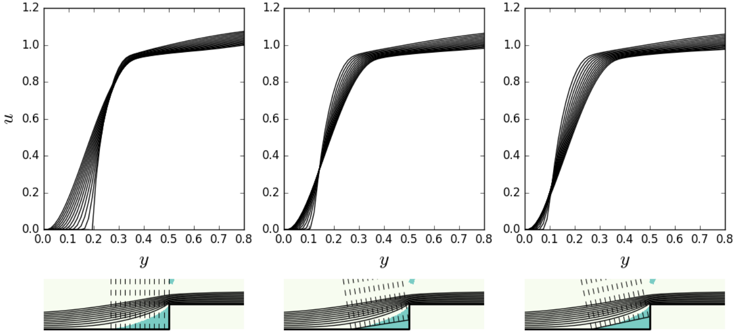




On the left, streamlines, dead and plug zones and probe lines (dashed dark lines) for the velocity profiles shown on the right for $B = 25$.

We retrieve the existence of the slip line !

Consistency with variations of θ



Velocity profiles for different tilted frames for $B = 25$.
The existence of the slip line is independant of θ.

What is the velocity profile above this slip line ?

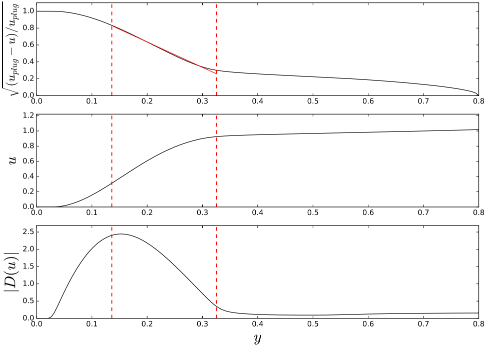
Goal : Show that the profile is in a certain sense Poiseuille-like above this slip line.

If we leave out the part below y_s , and suppose we have the slip velocity u_s, the Poiseuille flow becomes :

$$\left(\frac{u_{plug} - u(y)}{u_{plug} - u_s}\right)^{1/2} = \begin{cases} 1 - \dfrac{y - y_s}{y_{plug} - y_s} \text{ if } 0 \leqslant y \leqslant y_{plug} \\ 0 \text{ if } y > y_{plug}. \end{cases} \quad (4)$$

Hence, we perform a linear fit of the left hand side of (4) as a function of y to check whether it is really a Poiseuille.

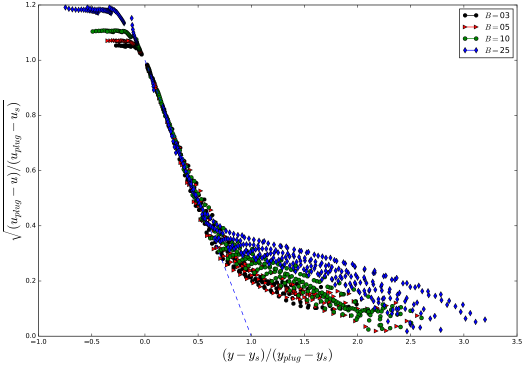
Numerical results - (1)



Left : Example for a particular cut for $B = 25$.
From top to bottom : $\sqrt{\frac{u_{plug}-u}{u_{plug}}}$, u and $|d|$.
We find good adequation between Poiseuille theory and results.

The red dashed lines represent respectively y_s and the end of the linear fit (determined manually)

Numerical results - (2)



Representation of $\sqrt{\frac{u_{plug} - u}{u_{plug} - u_s}}$ as a function of $\frac{y - y_s}{y_{plug} - y_s}$ for all cuts and for different B.

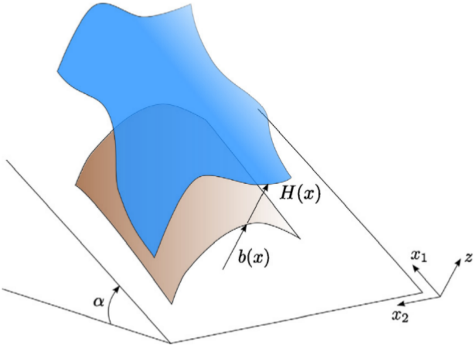
Every profile collapse on the same line, satisfying equation (4) !

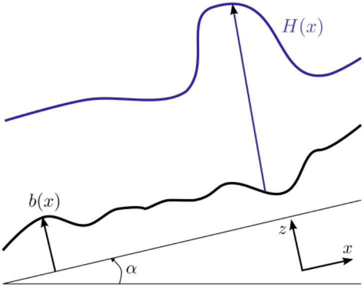
Outline

Résumé de l'épisode précédent

Ecole EGRIN no 2 en 2014 : le cas 1D pour un prototype

de modèle de Saint-Venant-Bingham (Hyp : vitesse(z)=cte)





Reminder : the 2D model, equation on V

$$\forall \boldsymbol{\Psi},\ \int_\Omega H\overline{\rho}\Big(\partial_t \boldsymbol{V}\cdot(\boldsymbol{\Psi}-\boldsymbol{V}) + \boldsymbol{V}\cdot\nabla_x \boldsymbol{V}(\boldsymbol{\Psi}-\boldsymbol{V})\Big)dX$$
$$+\int_\Omega \beta \boldsymbol{V}\cdot(\boldsymbol{\Psi}-\boldsymbol{V})dX$$
$$+\int_\Omega \frac{2}{\mathrm{Re}} H\eta D(\boldsymbol{V}) : D(\boldsymbol{\Psi}-\boldsymbol{V})dX$$
$$+\int_\Omega \frac{2}{\mathrm{Re}} H\eta \mathrm{div}_x \boldsymbol{V}(\mathrm{div}_x\boldsymbol{\Psi}-\mathrm{div}_x\,\boldsymbol{V})dX$$
$$+\int_\Omega \tau_y \mathrm{B} H\Big(\sqrt{|D(\boldsymbol{\Psi})|^2+(\mathrm{div}_x\boldsymbol{\Psi})^2}-\sqrt{|D(\boldsymbol{V})|^2+(\mathrm{div}_x\,\boldsymbol{V})^2}\Big)dX$$
$$\geq \frac{1}{\mathrm{Fr}^2}\int_\Omega H\overline{\rho\mathbf{F}_X}\cdot(\boldsymbol{\Psi}-\boldsymbol{V})dX - \frac{1}{\mathrm{Fr}^2}\int_\Omega H^2\overline{Z\rho F_z}(\mathrm{div}_x\boldsymbol{\Psi}-\mathrm{div}_x\boldsymbol{V})dX. \tag{5}$$

Rk1 : $\tau_y = 0$: 2D viscous SW *à la* Gerbeau-Perthame
Rk2 : for more details on model derivation $\rightarrow$ Bresch et al. Advances in Math. Fluid Mech. pp 57-89. 2010

Reminder : numerical schemes in 1D

Key ideas :

- First : Decouple the problem in H^{n+1} and V^{n+1}
- Problem in V^{n+1} : use a duality method (AL or BM)♣
- Problem in H^{n+1} & space discretization are linked :
  - underlying problem is a Viscous Shal. Water $\rightarrow$ F.V.
  - with source terms (including duality ones)
  - $\rightarrow$ need to design new Well-Balanced VF scheme
  - crucial to compute arrested state !

- Special treatment of viscoplastic wet/dry fronts
- ♣ Carefull study of optimal param., including 'a priori'

Synthetic Movie in 1D

Fernandez-Nieto, Gallardo, V. JCP 2014

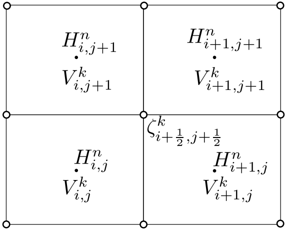
New stuff

We extend all the previous features in 2D :

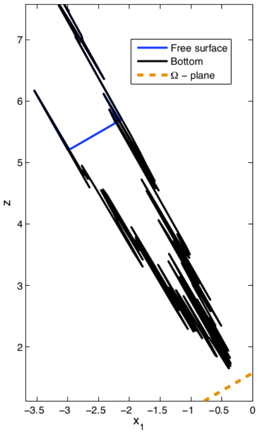
for conciseness not described here, but in short

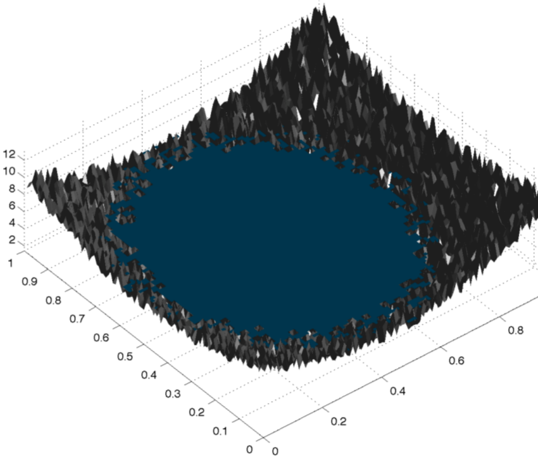
we did that on structured MAC grids to follow more easily

the link between $\boldsymbol{V}$ and dual variables ($\zeta \sim D(\boldsymbol{V})$)



2D WB test / Random bottom : initial condition





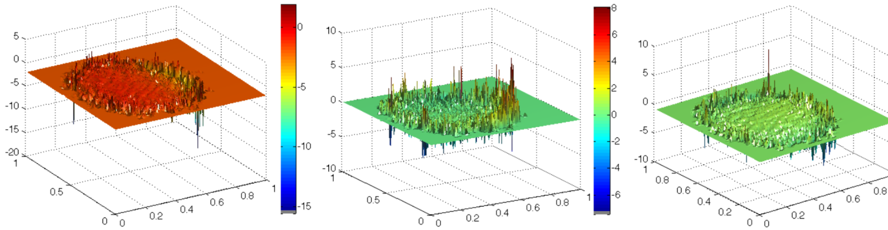
Slope $\alpha = 30^o$

If initialized with ζ from Theorem $\Rightarrow$ $V = 0$(machine precision).

2D WB test / Random bottom : duality multiplier

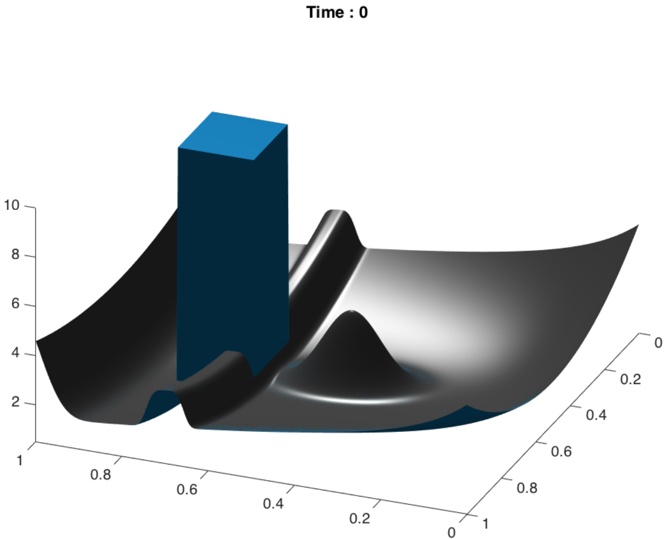
If initialized with $\zeta = 0$: accurate stationary state. 100^2 mesh

At $t = 1$. Left : ζ_{11}^k, center : ζ_{12}^k, right : ζ_{22}^k

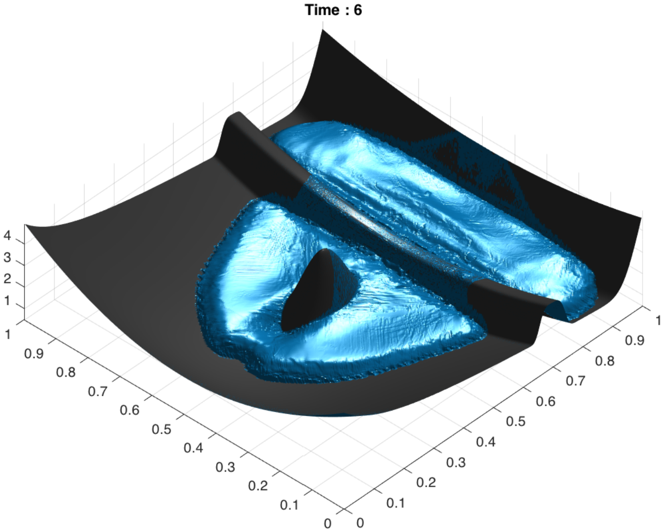


but ζ is computed at the first Δt and is then stationary on $[\Delta t, 1]$.

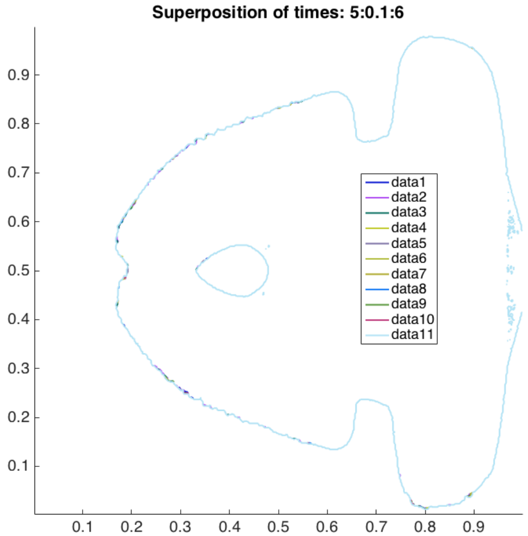
2D avalanche : initial condition and movie



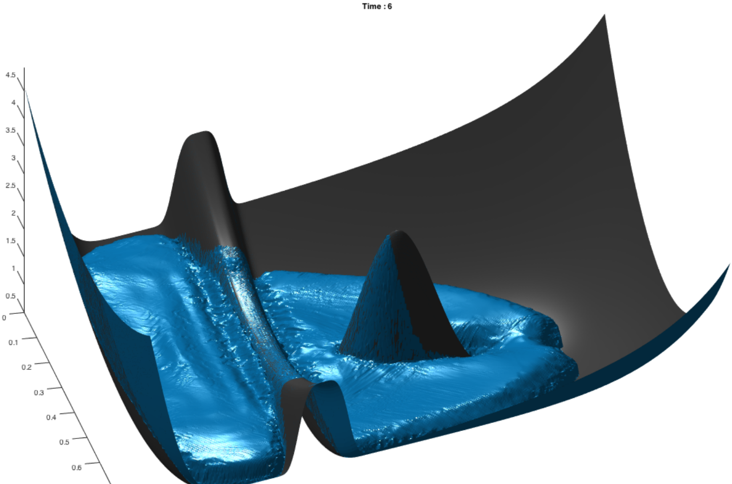
2D avalanche : t=6 and stationnary



2D avalanche : t=6 and stationnary {H=0}



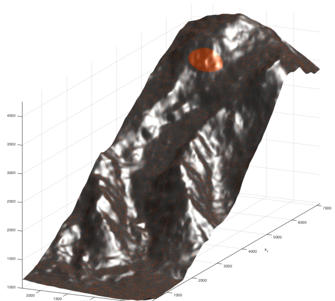
2D avalanche : t=6, high gradient !



Chamonix

A test with typical numerical difficulties of geophysical cases

- Domain : $\sim$ 2500 m $\times$ 7000 m
- Mesh : 210 $\times$ 430
- Topo : tortured, coming from satellite DEM
- Initial volume of "Gaussian break" : 0.6 $\times$ 10^6 m^3
- Rich dynamics, 3 phases : fast & medium before arrested



Conclusion

Achieved in Tellus 2016 : 3 papers published, 1 submitted

- Pushing limits of simulations of full 2D Bingham in cavities
- Bingham alone allows to retrieve many non trivial qualitative features of HB fluids
- Extension of SWB well-balanced schemes in 2D done
- Careful study of the associated duality methods, in particular viz. optimal parameters

Currently :

- Going further with the comparison experiments/simu in cavities : HB, etc
- Funded by CNRS Défi Interdisciplinaire InFIniti 2017-2018

Complementary bibliography

For more details and references, please see :

LH. Luu, P. Philippe, G. Chambon : Experimental study of the solid-liquid interface in a yield-stress fluid flow upstream of a step - Physical Review E **91**(1) - 2015

A. Marly, P. Vigneaux : Augmented Lagrangian simulations study of yield-stress fluid flows in expansion-contraction and comparisons with physical experiments - Journal of Non Newtonian Fluid Mechanics **239** : 35-52 - 2017

LH. Luu, P. Philippe, G. Chambon : Flow of a yield-stress fluid over a cavity: Experimental study of the solid–fluid interface - Journal of Non Newtonian Fluid Mechanics **245** : 25-37 - 2017

E.D. Fernández-Nieto, J.M. Gallardo, P. Vigneaux : Efficient numerical schemes for viscoplastic avalanches. Part 2 : the 2D case - 2017