Numerical modelling of tsunami mitigation by mangroves

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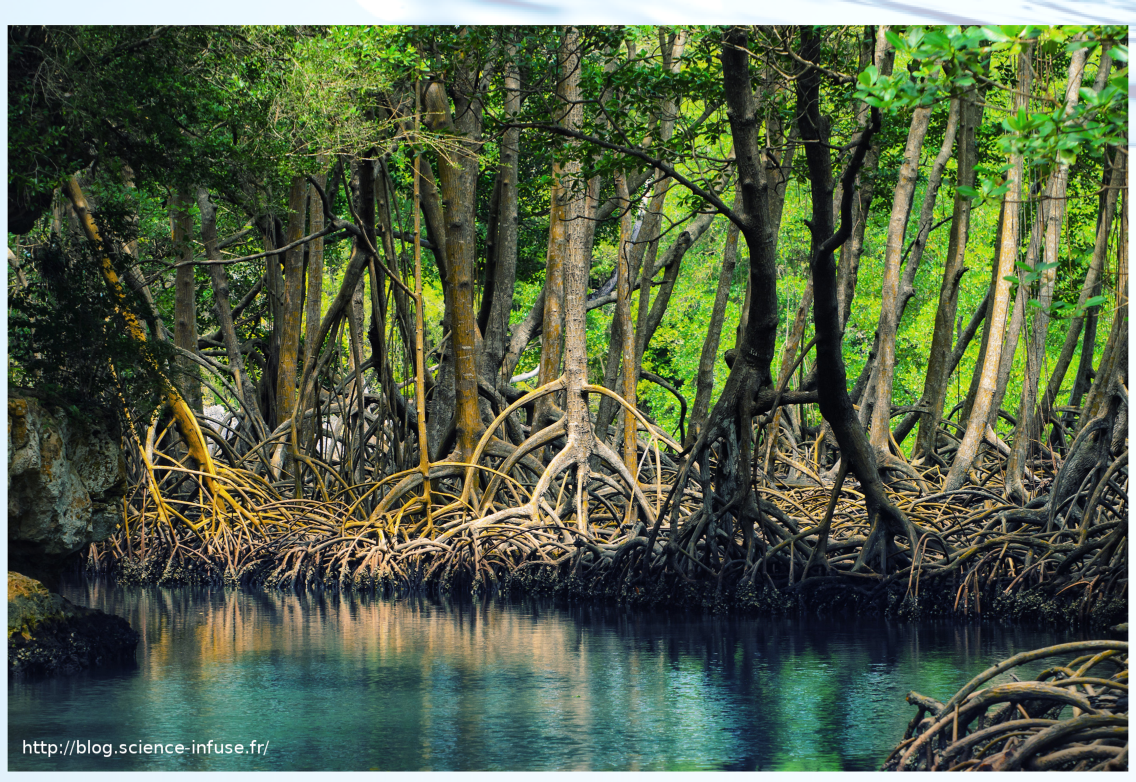
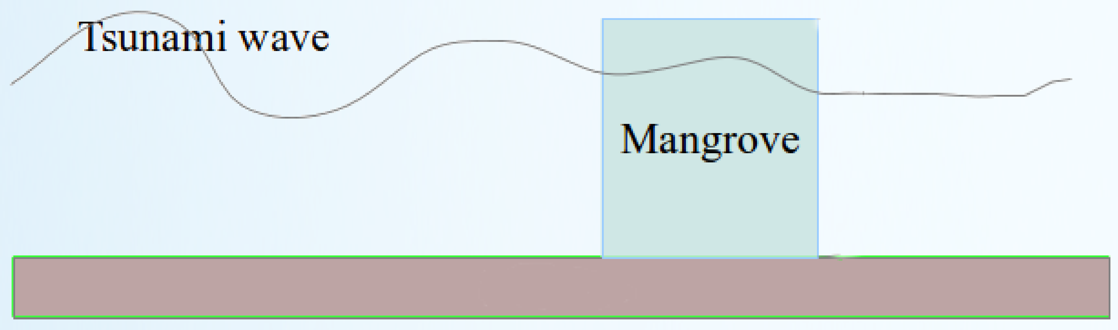
Problem



The role of mangroves (coastal forests) in the mitigation of tsunami impacts is a debated topic and numerical simulation can bring an important contribution to this debate. Several articles have been devoted to the numerical modelling of tsunami mitigation by mangroves in the past few years **[1, 2, 3]**. The arrival of a tsunami in a coastal area is generally described by the shallow water model.



Shallow water equations

A way of modelling the mangrove resistance is to add a bottom friction term (similar to those used in the simulation of overland flows **(Delestre)[4]**) to the shallow water equations:

$$\partial_t h + \partial_x(hu) = 0,$$
$$\partial_t(hu) + \partial_x\left(hu^2 + 1/2gh^2\right) + gh(\partial_x z + S_f) = 0,$$

where h is the water height, u is the velocity, g is the gravitational constant, z is the topography of the bottom, and S_f is a friction term due to the mangrove. A usual friction term is, for instance, the so-called Manning friction or Darcy-Weisbach:

$$S_f = C_f \frac{u|u|}{h^{4/3}}, \text{ or } S_f = C_f \frac{u|u|}{8gh},$$

where C_f is a given coefficient.

References

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[7] R. Herbin, et al, *Consistent explicit staggered schemes for compressible flows - Part I: the barotropic Euler equations*, submitted, 2013.

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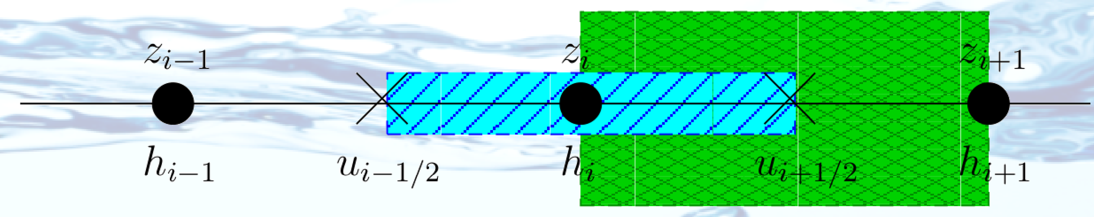
Numerical method



Generally, all the unknowns of the system are approximated on the same mesh and the numerical fluxes are computed with an **approximate Riemann solver**. We refer to **Bouchut [5] and Toro [6]** for a thorough description and analysis of this approach. **Staggered finite volume discretizations** for solving nonlinear hyperbolic system of conservation laws have been investigated more recently **Herbin et all [7] and Doyen et all [8].** The numerical fluxes can then be computed simply componentwise, using upwind or centered approximations.

We consider the time interval $(0,T)$ and the space domain $\Omega := (0,L)$ with solid wall boundary conditions (i.e. $u = 0$ at each end of the domain Ω). The time interval is divided into N_t time steps of length Δt and, for all $n \in \{0,\dots,N_t\}$, $t^n := n\Delta t$. The domain Ω is divided into N_x cells of length Δx. The left end, the center and the right end of the i-th cell are denoted by $x_{i-\frac12}$, x_i and $x_{i+\frac12}$, respectively. We set $\mathcal{M} := \{1,\dots,N_x\}$, $\mathcal{E}_{int} := \{1,\dots,N_x-1\}$, $\mathcal{E}_b := \{0,N_x\}$, and $\mathcal{E} := \mathcal{E}_{int} \cup \mathcal{E}_b$. The mass conservation equation is discretized with an explicit upwind scheme:

$$h_i^{n+1} - h_i^n + \frac{\Delta t}{\Delta x}\left(q^n_{i+\frac12} - q^n_{i-\frac12}\right) = 0, \qquad \forall i \in \mathcal{M},$$

where

$$q^n_{i+\frac12} := \hat h^n_{i+\frac12} u^n_{i+\frac12}, \qquad \hat h^n_{i+\frac12} := \begin{cases} h_i^n & \text{if } u^n_{i+\frac12} \geq 0 \\ h^n_{i+1} & \text{if } u^n_{i+\frac12} < 0 \end{cases}, \qquad \forall i \in \mathcal{E}.$$

The momentum balance equation is discretized with explicit upwind fluxes for the convection term and implicit centered fluxes for the pressure term and topography term:

$$h^{n+1}_{i+\frac12}u^{n+1}_{i+\frac12} - h^n_{i+\frac12}u^n_{i+\frac12} + \frac{\Delta t}{\Delta x}\Big[q^n_{i+1}\hat u^n_{i+1} - q^n_i \hat u^n_i + \frac12 g\left[(h^{n+1}_{i+1})^2 - (h^{n+1}_i)^2\right] + g h^{n+1}_{i+\frac12}(z_{i+1}-z_i)\Big] + \frac{\Delta t g C_f}{\left(h^{n+1}_{i+\frac12}\right)^{1/3}}|u^n_{i+\frac12}|u^{n+1}_{i+\frac12} = 0, \qquad \forall i\in\mathcal{E}_{int},$$

where

$$h^n_{i+\frac12} := \frac12\left(h_i^n + h^n_{i+1}\right), \qquad \forall i \in \mathcal{E}_{int},$$

$$q_i^n := \frac12\left(q^n_{i-\frac12} + q^n_{i+\frac12}\right), \qquad \hat u_i^n := \begin{cases} u^n_{i-\frac12} & \text{if } q_i^n \geq 0 \\ u^n_{i+\frac12} & \text{if } q_i^n < 0 \end{cases}, \qquad \forall i \in \mathcal{M}.$$

At each time step, the Courant number is defined by

$$\nu := \frac{\Delta t}{\Delta x}\max_{i\in\mathcal{M}}\left(\frac{\left|q^n_{i+\frac12} + q^n_{i-\frac12}\right|}{2h_i^n} + \sqrt{gh_i^n}\right).$$

The numerical simulations show that the staggered scheme is stable under the CFL condition $\nu \leq 1$.

Numerical simulations