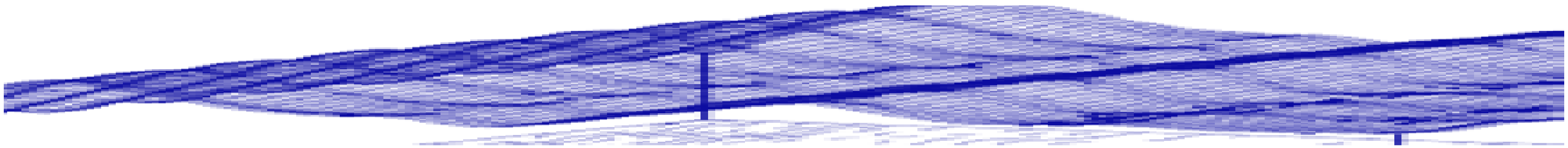
i3m Institut de Mathématiques et de Modélisation de Montpellier

CERFACS



Extreme scenarios for the evolution of a soft bed interacting with a fluid using the VaR of the bed characteristics

UNIVERSITÉ MONTPELLIER 2 SCIENCES ET TECHNIQUES

CNRS

Summary

Contexte : Optimization under uncertainties.
Application : Morphodynamics by minimization principle.

-VaR-based extreme scenarios.
Also used to provide worst-case scenarios in robust optimization.

- Bed motion and its local variability linked through an original transport equation.

Contexte : Robust optimization with randomness well located in a simulation chain.

$$x \longrightarrow X(= x + \varepsilon(x)) \longrightarrow u(X) \longrightarrow j(u(X))$$

PDF of $\varepsilon(x)$ known and in particular its VaR

Want to avoid any sampling of the parameter space and propagation of the uncertainty in the simulation chain.

Introduce VaR-based worst-case scenarios in a deterministic minimization algorithm.

Application :

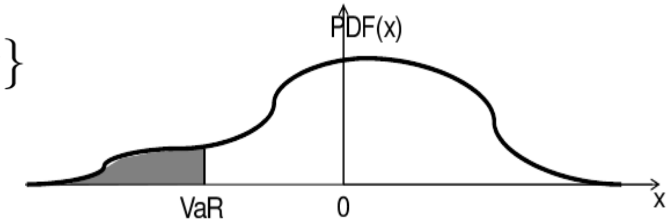
Aerodynamic shape design,

Design of defense structures against littoral erosion

VaR

In finance, it defines for a given probability level and time horizon a threshold value for the loss X.

$$\mathrm{VaR}_\alpha = \inf\{l \in \mathbb{R} : P(X > l) \leqslant 1 - \alpha\}$$



Knowing the PDF of the uncertainties $\varepsilon(x)$ on a control parameter x, define two VaR-based extreme scenarios for the variations:

$X = x + \mathrm{VaR}^{\alpha}_{\pm}(x)$, with $\mathrm{VaR}^{\alpha}_{-} \leq 0 \leq \mathrm{VaR}^{\alpha}_{+}$ (defined component by component)

If Gaussian PDF:

$\mathrm{VaR}^{0.99} = 2.33$ and $\mathrm{VaR}^{0.95} = 1.65$ for N(0,1)

and $\mathrm{VaR}^{\alpha}(\mathrm{N}(0, \sigma(x))) = \sigma(x)\ \mathrm{VaR}^{\alpha}(\mathrm{N}(0,1))$

and $\mathrm{VaR}^{\alpha}_{-} = -\mathrm{VaR}^{\alpha}_{+}$

Deterministic model

Dependency chain: $\psi \rightarrow \{\mathbf{U}(\psi,\tau), \tau \in [0,T]\} \rightarrow J(\psi,T)$

Flow state equation: $\mathbf{U}_t + F(\mathbf{U},\psi) = 0, \quad \mathbf{U}(0) = \mathbf{U}_0(\psi)$

Cost fct: $J(\psi,T) = \int_{(0,T)} j(\psi, \mathbf{U}(\psi,t))$

Bed motion: $\partial_t \psi = -\rho(t,x)\nabla_\psi J, \qquad \psi(t=0,x) = \psi_0(x) =$ given,

Linearizing J: $J_\psi(\psi,T) = \int_{(0,T)} (j_\psi + j_{\mathbf{U}} \mathbf{U}_\psi)$

Introduce here variability on bed characteristics. This is non intrusive for the heavy simulation tools.

Linearized state equation:

$(\mathbf{U}_\psi)_t + F_\psi(\mathbf{U},\psi) + F_{\mathbf{U}}(\mathbf{U},\psi)\mathbf{U}_\psi = 0, \quad \mathbf{U}_\psi(0) = \mathbf{U}_0'(\psi)$

$\forall \mathbf{V} = (v_1, \mathbf{v_2})^t$ (same structure than $\mathbf{U} = {}^t(h, h\mathbf{u})$) one has:

$$0 = \int_{(0,T)\times\Omega} ((\mathbf{U}_\psi)_t + F_\psi(\mathbf{U},\psi) + F_{\mathbf{U}}(\mathbf{U},\psi)\mathbf{U}_\psi)\ \mathbf{V}$$

$$0 = \int_{(0,T)\times\Omega} (-\mathbf{V}_t + F_{\mathbf{U}}^*(\mathbf{U},\psi)\ \mathbf{V})\mathbf{U}_\psi + \int_\Omega [\mathbf{V}\mathbf{U}_\psi]_0^T + \int_{(0,T)\times\Omega} \mathbf{V} F_\psi(\mathbf{U},\psi)$$

Backward adjoint problem: $\mathbf{V}_t + F_{\mathbf{U}}^*(\mathbf{U},\psi) = j_{\mathbf{U}}, \qquad \mathbf{V}(T) = 0$

$$\int_{(0,T)\times\Omega} j_{\mathbf{U}} \mathbf{U}_\psi = \int_\Omega \mathbf{V}(0)\mathbf{U}_0'(\psi) - \int_{(0,T)\times\Omega} \mathbf{V} F_\psi(\mathbf{U},\psi)$$

For SVE, dependency in ψ is in $gh\nabla\psi$:

$$\int_{(0,T)\times\Omega} \mathbf{V} F_\psi(\mathbf{U},\psi) = -\int_{(0,T)\times\Omega} g\nabla.(h\mathbf{v_2})$$

Example of functional

for beach morphodynamics simulations
(MathOcean and Copter ANR)

T: Time interval of influence
Ω *:* observation domain

$$J(U(\psi)) = \int_{t-T}^{t} \int_{\Omega} (\frac{1}{2} \rho_w g \eta^2 + \rho_s g (\psi(\tau) - \psi(t-T))^2) d\tau d\Omega$$

$$\eta(\tau, x, \psi) = h(\tau, x, \psi) - \frac{1}{T} \int_{t-T}^{t} h(\tau, x, \psi) d\tau$$

Hypothesis:

The bed adapts in order to reduce water kinetic energy with 'minimal' sand transport.

Approach can be seen as an Exner equation with nonlocal flux term.

Extreme scenarios

Knowing the PDF of the bed characteristics and given a confidence level, provide extreme evolution scenarios for the bed at a computational cost comparable to a single simulation (no sampling of the parameter space).

VaR

Knowing the PDF of the uncertainties on the bed receptivity, define two extreme scenarios for our fluid-structure coupling with (*x* space coordinates)

$$0 \leq \rho(x) = \rho_0(x) + \mathrm{VaR}_{\pm}^{\alpha}(x), \text{ with } \mathrm{VaR}_{-}^{\alpha} \leq 0 \leq \mathrm{VaR}_{+}^{\alpha}$$

…To go beyond stationarity of the variability and therefore of the bed receptivity.

Extreme scenarios for a bed adapting to a flow

$\rho_0 = 0.0002 m/s$ and $0.002 m/s$

$VaR^{\pm}_{0.95}(N(0,\sigma(x)))$, with $\sigma(x)$ linearly decreasing cross-shore

$\psi(t=0,x) = \text{flat}$

Water wave elevation is addition of monochromatic waves or Jonswap.

$\text{T} = 5\text{s}$

Water height at rest $0.7m$

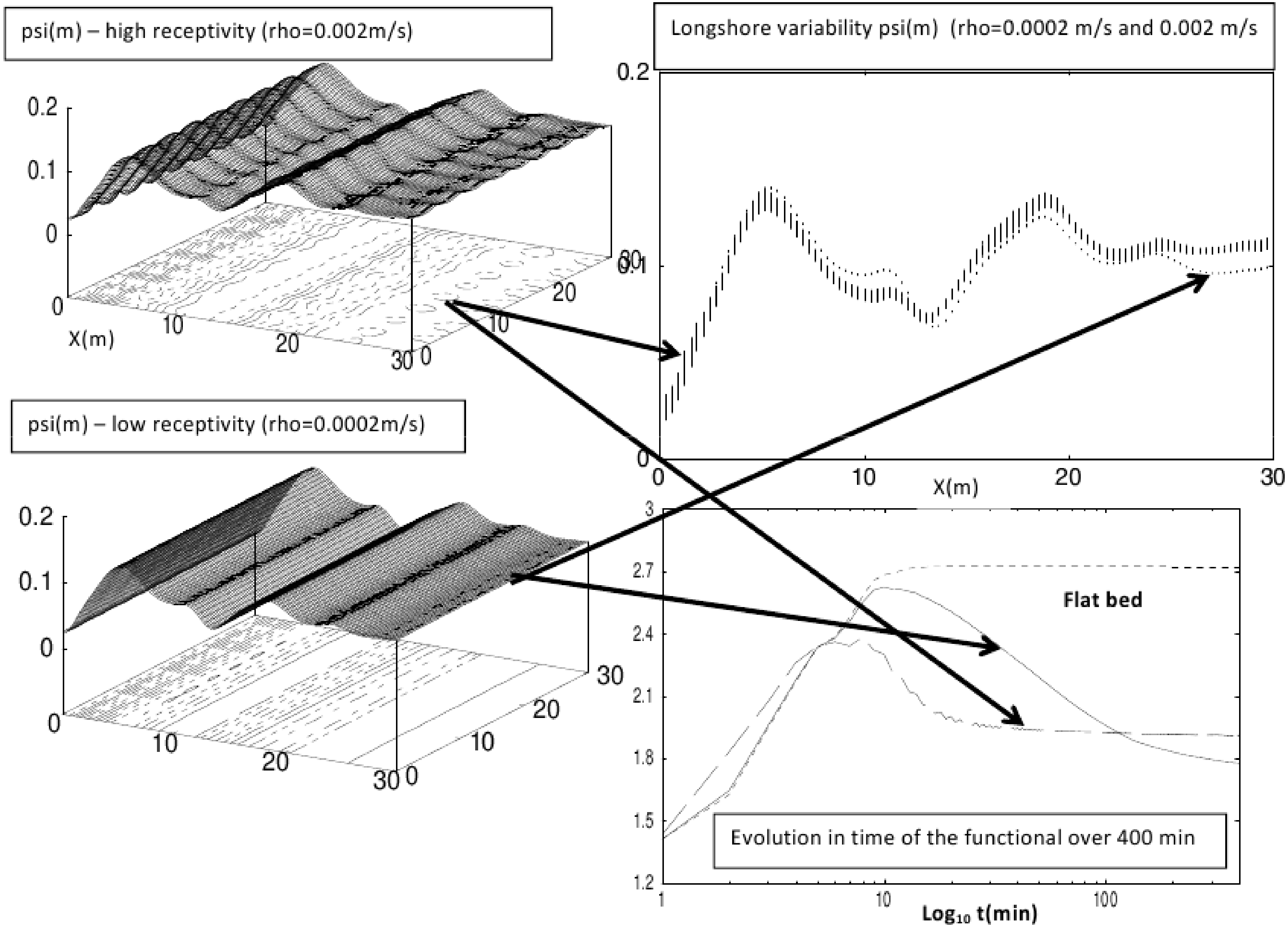
Domain is $30m$ x $30m$

(Sogreah basin, Grenoble H. Michalet/F.Bouchette, ANR Copter)

Find a shape removing water elevation and its gradient.:

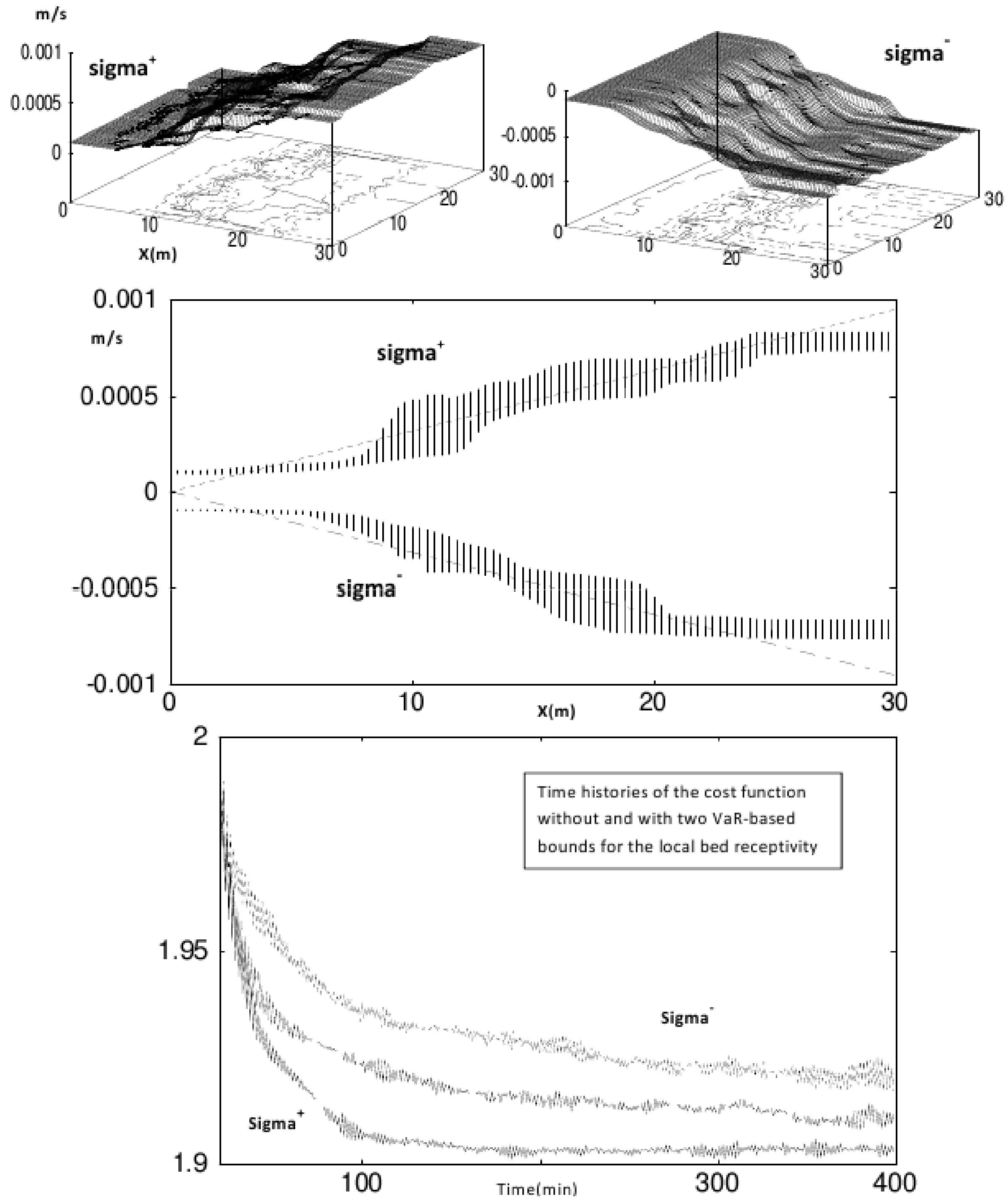
$$J(\psi) = \frac{1}{|\Omega| T} \int_{t-T}^{t} \int_{\Omega} |\eta| \|\nabla_x \eta\| \; d\tau d\Omega,$$

Without bed receptivity variability



0,002m/s receptivity
+ PDF

Two extreme VaR-based scenarios



Remarks

- Morphodynamics by minimization principle.

-VaR-based extreme scenarios.

- Quantify confidence level on bed evolution without any sampling of the bed characteristics.

- Bed motion and its local variability linked through an original transport equation.

- Need to account for bed variability during the coupling and not only eventually through engineering margins.