A multilayer approach for sediment transport in shallow water regime

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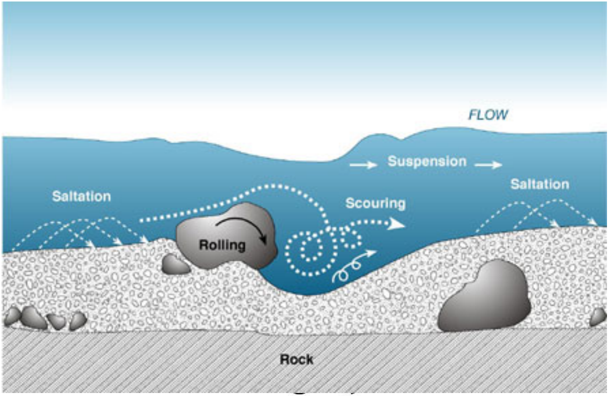
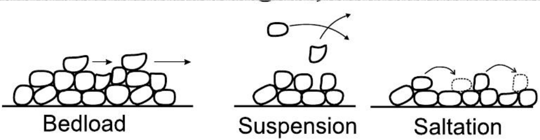
UNIVERSIDAD DE CÓRDOBA

Grupo EDANYA

Outline

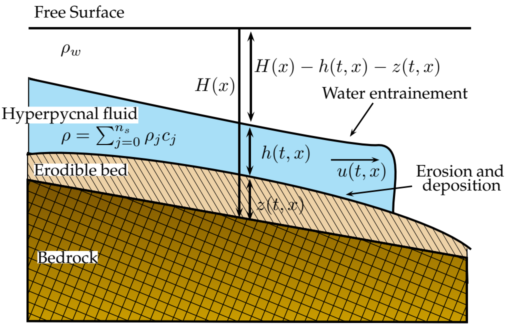
Outline

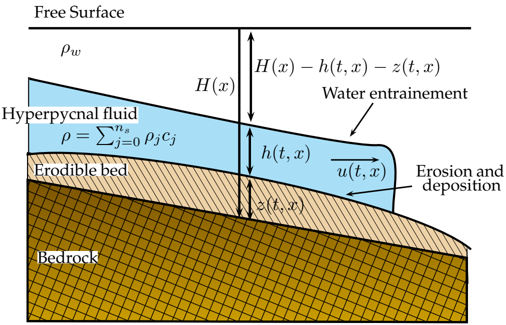

FLOW
Suspension
Saltation
Saltation
Scouring
Rolling
Rock


Bedload
Suspension
Saltation

- Profound impact on the morphology of continental shelves
- Deposit $\Rightarrow$ porous layer of rock $\Rightarrow$ potential sources of hydrocarbon
- Destructive effect (pipelines, cables, fundations,...)

Turbidity current / Hyperpicnal plume





$$
\begin{cases}
\partial_t h + \partial_x(hu) = \phi_\eta + \phi_b, \\
\partial_t(hu) + \partial_x\left(hu^2 + g\,(R_0 + R_c)\,\frac{h^2}{2}\right) = \\
\qquad\qquad g\,(R_0 + R_c)\,h\partial_x(H - z) + u\phi_\eta + \frac{u}{2}\phi_b + \tau, \\
\partial_t(hc_j) + \partial_x(hu\,c_j) = \phi_b^j, \ \text{for } j = 1, \ldots, n_s \\
\partial_t z + \xi\partial_x q_b = -\xi\phi_b.
\end{cases}
$$

Hyperpycnal model

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\end{cases}
\qquad \text{(H)}
$$

$$
R_j = \frac{\rho_j - \rho_0}{\rho_0}, \ \text{for } j = 1, \ldots, n_s; \quad R_0 = \frac{\rho_0 - \rho_w}{\rho_0}; \quad \text{and } R_c = \sum_{j=1}^{n_s} R_j c_j.
$$

Hyperpycnal model

$$
\left\{
\begin{array}{l}
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\end{array}
\right. \tag{H}
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Water entrainement

$$\phi_\eta = E_w u,$$

$$E_w = \frac{0.00153}{0.0204 + \mathcal{R}i}, \qquad \mathcal{R}i = \frac{R_c gh}{u^2}.$$

Hyperpycnal model

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\end{cases} \tag{H}
$$

Erosion/deposition

$$
\phi_b^j = F_e^j - F_d^j, \quad \phi_b = \sum_{j=1}^{n_s} \phi_b^j
$$

Areas of different composition

Applications

(Controlled) sedimentation of polydisperse suspensions of small particles
N species differing in size d_i and density ρ_i

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N species differing in size d_i and density ρ_i

- Chemical engineering
- Industrial procesing
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- Etc.

Outline

1. Introduction

2. Polydisperse sedimentation
   - Masliyah-Lockett-Bassoon (MLB) model

3. A multilayer approach
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Polydisperse sedimentation model

N especies of sedidemnt
ϕ_i = volumetric concentration $i = 1, \ldots, N$

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$$\phi = \sum_{i=1}^{N} \phi_i, \quad \phi_0 = 1 - \phi, \text{ and } \Phi = (\phi_0, \phi_1, \ldots, \phi_N)$$

$$\rho(\Phi) = \sum_{i=0}^{N} \rho_i \phi_i$$

Local mass balance equations

$$\partial_t \phi_i + \nabla \cdot (\phi_i v_i) = 0, \quad i = 0, 1, \ldots, N,$$

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Average velocity

$$q = \sum_{i=0}^{N} \phi_i v_i$$

Relative/slip velocity

$$\Delta v_i = v_i - v_0, \quad i = 1, \ldots, N$$

Local mass balance equations

$$\partial_t \phi_i + \nabla \cdot (\phi_i v_i) = 0, \quad i = 0, 1, \ldots, N,$$

Local mass balance equations

$$\partial_t \phi_i + \nabla \cdot \left(\phi_i \Delta v_i + \phi_i q - \phi_i \sum_{j=1}^{N} \phi_j \Delta v_j \right), \quad i = 0, 1, \ldots, N,$$

Local mass balance equations

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Sum of all equations:

$$\nabla \cdot q = 0$$

Momentum balance equations

$$\rho_i(\partial_t(\phi_i v_i) + \nabla \cdot (\phi_i v_i \otimes v_i)) \\ = -\rho_i \phi_i g \vec{k} - \nabla p_i + \nabla \cdot T_i^E + m_i^f + m_i^s, \quad i = 1, \ldots, N,$$

$$\rho_0(\partial_t(\phi_0 v_0) + \nabla \cdot (\phi_0 v_0 \otimes v_0)) \\ = -\rho_0 \phi_0 g \vec{k} - \nabla p_0 + \nabla \cdot T_0^E - (m_1^f + \ldots + m_N^f),$$

Momentum balance equations

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m_i^f interaction forces between solid especie i and fluid

Momentum balance equations

$$\rho_i(\partial_t(\phi_i v_i) + \nabla\cdot(\phi_i v_i \otimes v_i)) \\ = -\rho_i\phi_i g\vec{k} - \nabla p_i + \nabla\cdot T_i^E + m_i^f + m_i^s, \quad i = 1,\ldots,N,$$

$$\rho_0(\partial_t(\phi_0 v_0) + \nabla\cdot(\phi_0 v_0 \otimes v_0)) \\ = -\rho_0\phi_0 g\vec{k} - \nabla p_0 + \nabla\cdot T_0^E - (m_1^f + \ldots + m_N^f),$$

$m_i^s = m_{i1}^s + \ldots + m_{iN}^2$,
m_{ij}^2 interaction forces between solid especies i and j

Momentum balance equations

$$\rho_i(\partial_t(\phi_i v_i) + \nabla \cdot (\phi_i v_i \otimes v_i)) \\ = -\rho_i \phi_i g \vec{k} - \nabla p_i + \nabla \cdot T_i^E + m_i^f + m_i^s, \quad i = 1, \ldots, N,$$

$$\rho_0(\partial_t(\phi_0 v_0) + \nabla \cdot (\phi_0 v_0 \otimes v_0)) \\ = -\rho_0 \phi_0 g \vec{k} - \nabla p_0 + \nabla \cdot T_0^E - (m_1^f + \ldots + m_N^f),$$

T_i^E viscous stress tensors

Momentum balance equations

$$\rho_i(\partial_t(\phi_i v_i) + \nabla\cdot(\phi_i v_i \otimes v_i)) = -\rho_i\phi_i g\vec{k} - \nabla p_i + \nabla\cdot T_i^E + m_i^f + m_i^s, \quad i = 1,\dots,N,$$

$$\rho_0(\partial_t(\phi_0 v_0) + \nabla\cdot(\phi_0 v_0 \otimes v_0)) = -\rho_0\phi_0 g\vec{k} - \nabla p_0 + \nabla\cdot T_0^E - (m_1^f + \ldots + m_N^f),$$

p_i phase pressure

Momentum balance equations

$$p_t = p_0 + p_1 + \ldots + p_N = p + \sigma_e(\phi)$$

Momentum balance equations

$$p_t = p_0 + p_1 + \ldots + p_N = p + \sigma_e(\phi)$$

p_t total stress, p pore pressure, σ_e effective solid stress

Momentum balance equations

$$p_t = p_0 + p_1 + \ldots + p_N = p + \sigma_e(\phi)$$

$$\phi := \phi_1 + \ldots \phi_N < \phi_c \Rightarrow \sigma_e = 0$$

Momentum balance equations

$$p_t = p_0 + p_1 + \ldots + p_N = p + \sigma_e(\phi)$$

$$p_i = \frac{\phi_i}{\phi} \left(\phi p + \sigma_e(\phi) \right)$$

Momentum balance equations

$$m_i^f = \alpha_i(\Phi)\Delta v_i + \beta_i(\Phi)\nabla\phi_i$$

α_i resistence coefficient

$$m_{ij}^s = \frac{3}{2}\phi_e \frac{\rho_i\rho_j\phi_i\phi_j(d_i + d_j)}{\rho_i d_i^3 + \rho_j d_j^3}||v_i - v_j||(v_i - v_j)$$

Polydisperse sedimentation model

$$\partial_t \phi_i + \nabla(\phi_i v_i) = 0, \quad i = 0, 1, \ldots, N,$$

$$\rho_i(\partial_t(\phi_i v_i) + \nabla \cdot (\phi_i v_i \otimes v_i))$$
$$= -\rho_i \phi_i g \vec{k} - \phi_i \nabla p + \alpha_i(\phi) \Delta v_i + m_i^s + \nabla \cdot T_i^E - \nabla \left(\frac{\phi_i}{\phi} \sigma_e(\phi) \right), \; i = 1, \ldots, N,$$

$$\rho_0(\partial_t(\phi_0 v_0) + \nabla \cdot (\phi_0 v_0 \otimes v_0)) = -\rho_0 \phi_0 g \vec{k} \sum_{i=1}^{N} \alpha_i(\phi) \Delta v_i + \nabla \cdot T_0^E,$$

$v_i = (u_i, w_i) \in \mathbb{R}^2$ is the phase velocity
T_i^E viscous stress tensor
m_i^s interaction forces between solid particles
σ_i^E effective solid stress
α_i resistence coefficient for the transfer of momentum

Polydisperse sedimentation model

$$\partial_t \phi_i + \nabla(\phi_i v_i) = 0, \quad i = 0, 1, \ldots, N,$$

$$\rho_i(\partial_t(\phi_i v_i) + \nabla \cdot (\phi_i v_i \otimes v_i))$$
$$= -\rho_i \phi_i g \vec{k} - \phi_i \nabla p + \alpha_i(\phi)\Delta v_i + m_i^s + \nabla \cdot T_i^E - \nabla \left(\frac{\phi_i}{\phi} \sigma_e(\phi) \right), \; i = 1, \ldots, N,$$

$$\rho_0(\partial_t(\phi_0 v_0) + \nabla \cdot (\phi_0 v_0 \otimes v_0)) = -\rho_0 \phi_0 g \vec{k} \sum_{i=1}^{N} \alpha_i(\phi)\Delta v_i + \nabla \cdot T_0^E,$$

$$q = \sum_{i=0}^{N} \phi_i v_i$$

$$\Delta v_i = v_i - v_0, \quad i = 1, \ldots, N.$$

Sherman-Morrison formula

$$\Delta v_i = \frac{\phi}{\alpha_i(\Phi)} \left[(\rho_i - \rho(\Phi)) g \vec{k} + \frac{\sigma_e(\phi)}{\phi_i} \nabla \left(\frac{\phi_i}{\phi} \right) + \frac{1-\phi}{\phi} \nabla \sigma_e(\phi) \right]$$

Outline

Masliyah-Lockett-Bassoon

$$\frac{\phi}{\alpha_i(\Phi)} = -d_i^2 \frac{V(\Phi)}{18\mu_f}$$

μ_f viscosity of pure fluid
$V(\Phi) = (1-\phi)^{n-2}$, $(n > 2)$ hindered settling factor

MLB Model

$$\Delta v_i = \mu \delta_i V(\phi) \left((\overline{\rho}_i - \sum_{j=1}^{N} \overline{\rho}_j \phi_j)\vec{k} + \frac{\sigma_e(\phi)}{g\phi_i} \nabla \left(\frac{\phi_i}{\phi} \right) + \frac{1-\phi}{g\phi} \nabla \sigma_e(\phi) \right)$$

$$V(\phi) = (1-\phi)^{n-2},\ n > 2.$$

$$\overline{\rho_i} = \rho_i - \rho_0 \text{ for } i = 1, \dots, N,\ \mu = -g\frac{d_1^2}{18\mu_f}\ \delta_i = \frac{d_i^2}{d_1^2}$$

MLB Model

$$\phi_i v_i = f_i(\Phi)\vec{k} + \phi_i q - a_i(\Phi, \nabla\Phi)$$

$$f_i(\phi) = \mu V(\phi)\phi_i \left(\delta_i(\overline{\rho}_i - \sum_{j=1}^{N} \overline{\rho}_j \phi_j) - \sum_{k=1}^{N} \delta_k \phi_k \left(\overline{\rho}_k - \sum_{j=1}^{N} \overline{\rho}_j \phi_j \right) \right),$$

MLB model

Assuming $\partial_t(\phi_i w_i) + \partial_x(\phi_i w_i u_i) + \partial_z(\phi_i w_i^2)$ and T_i^E small

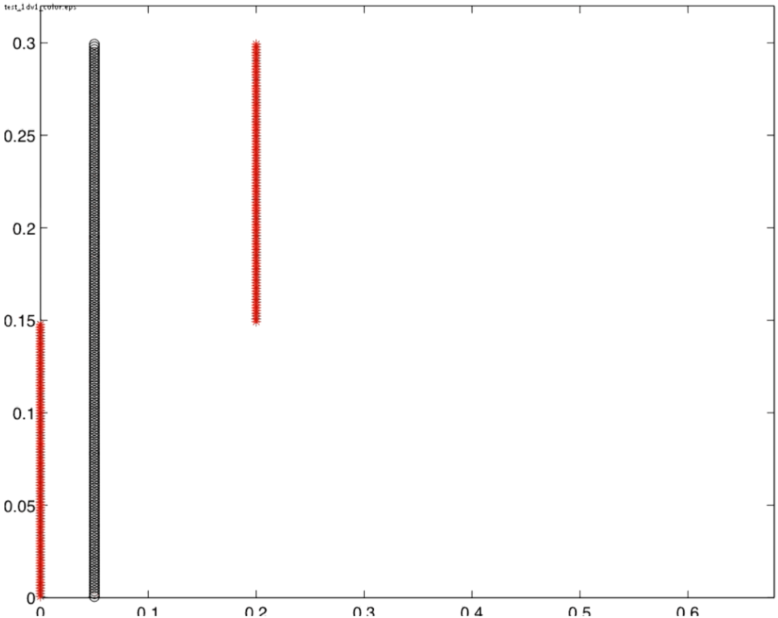
$$\begin{cases} \partial_t \phi_i + \partial_x(\phi_i u_i) + \partial_z(\phi_i w + f_i(\phi)) = \partial_z(a_i(\phi, \partial_z \phi)), \quad i = 1, \ldots, N \\ \nabla \cdot q = 0 \\ \nabla p = -\nabla \sigma_e(\phi) - \rho(\phi) g \vec{k} + \dfrac{1}{1-\phi} \nabla \cdot T_0^E \end{cases}$$

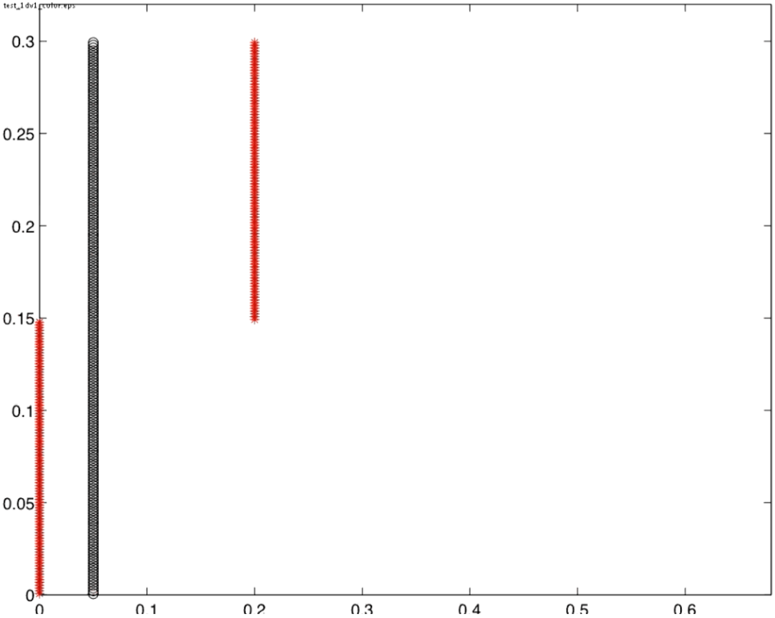
$$f_i(\phi) = \mu V(\phi) \phi_i \left(\delta_i (\overline{\rho}_i - \sum_{j=1}^N \overline{\rho}_j \phi_j) - \sum_{k=1}^N \delta_k \phi_k \left(\overline{\rho}_k - \sum_{j=1}^N \overline{\rho}_j \phi_j \right) \right),$$

MLB model for a one-dimensional closed vessel

$$\partial_t \phi_i + \partial_z (f_i(\phi)) = \partial_z (a_i(\phi, \partial_z \phi)), \quad i = 1, \ldots, N$$

$$f_i(\phi) = \mu V(\phi)\phi_i \left(\delta_i (\overline{\rho}_i - \sum_{j=1}^{N} \overline{\rho}_j \phi_j) - \sum_{k=1}^{N} \delta_k \phi_k \left(\overline{\rho}_k - \sum_{j=1}^{N} \overline{\rho}_j \phi_j \right) \right),$$


test_1dv1_color.eps
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5
0.6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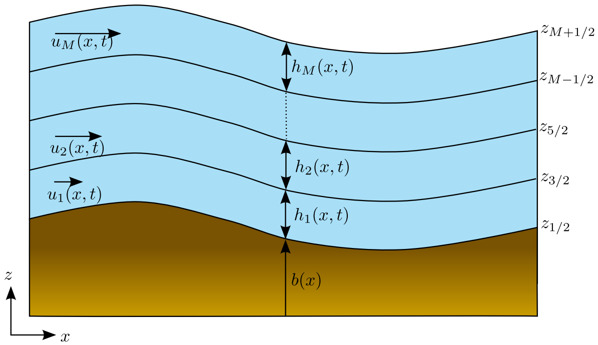
A multilayer approach



A multilayer approach

$$\begin{cases} \partial_t(h_\alpha \phi_{i,\alpha}) + \partial_x(h_\alpha \phi_{i,\alpha} u_\alpha) = H_{i,\alpha+\frac{1}{2}} - H_{i,\alpha+\frac{1}{2}}, \\ \partial_t h_\alpha + \partial_x(h_\alpha u_\alpha) = G_{\alpha+\frac{1}{2}} - G_{\alpha-\frac{1}{2}} - \displaystyle\sum_{i=0}^{N} \Big(f_{i,\alpha+\frac{1}{2}}(\Phi) - f_{i,\alpha-\frac{1}{2}}(\Phi) \Big), \\ \partial_t(\rho_\alpha h_\alpha u_\alpha) + \partial_x(\rho_\alpha h_\alpha u_\alpha^2 + h_\alpha(p_{\alpha+1/2} + g \sum_{i=0}^N \rho_i \phi_{i,\alpha} \frac{h_\alpha}{2})) \\ \qquad = \rho|_{z=z_{\alpha+1/2}} H_{i,\alpha+1/2} - \rho|_{z=z_{\alpha-1/2}} H_{i,\alpha-1/2} + \tau_\alpha \\ \qquad\quad + p_{\alpha+1/2} \partial_x z_{\alpha+1/2} - p_{\alpha-1/2} \partial_x z_{\alpha-1/2} \end{cases}$$

$$H_{i,\alpha+\frac{1}{2}} = \phi_{i,\alpha+\frac{1}{2}} G_{\alpha+\frac{1}{2}} - f_{i,\alpha+\frac{1}{2}}(\Phi)$$

$$G_{\alpha+\frac{1}{2}} = \partial_t z_{\alpha+\frac{1}{2}} + u_{\alpha+\frac{1}{2}} \partial_x z_{\alpha+\frac{1}{2}} - w_{\alpha+\frac{1}{2}}$$

For $\alpha = 1, \ldots, M$, $h_\alpha = l_\alpha h$ with l_α a positive constant.

$$\sum_{\alpha=1}^{M} l_\alpha = 1.$$

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$$\sum_{\alpha=1}^{M} l_\alpha = 1.$$

$$\partial_t h + \partial_x \left(h \sum_{\beta=1}^{M} l_\beta u_\beta \right) = G_{M+1/2} - G_{1/2}.$$

$$
\begin{cases}
\partial_t h + \partial_x \left(h \sum_{\beta=1}^{M} l_\beta u_\beta \right) = G_{M+1/2} - G_{1/2}, \\
\partial_t (h\phi_{j,\alpha}) + \partial_x (h\phi_{j,\alpha} u_\alpha) = \dfrac{1}{l_\alpha} \Delta_-^\alpha \big(H_{j,\alpha+1/2}(\Phi) \big), \\
\partial_t \big(h\rho_\alpha(\Phi) u_\alpha \big) + \partial_x \big(h\rho_\alpha(\Phi) u_\alpha^2 \big) \\
\quad + \partial_x \left(h p_{\mathrm{S}} + g h^2 \left[\dfrac{1}{2} l_\alpha \, \rho_\alpha(\Phi) \; + \; \sum_{\beta=\alpha+1}^{M} l_\beta \, \rho_\beta(\Phi) \right] \right) \\
\quad = \sum_{j=0}^{N} \dfrac{\rho_j}{l_\alpha} \Delta_-^\alpha \big(u_{\alpha+1/2} H_{j,\alpha+1/2}(\Phi) \big) + \left(p_{\mathrm{S}} + g h \sum_{\beta=\alpha+1}^{M} l_\beta \rho_\beta(\Phi) \right) \partial_x h \\
\qquad - g h \rho_\alpha \left(\partial_x z_{\mathrm{B}} + \sum_{\beta=1}^{\alpha-1} l_\beta \partial_x h \right).
\end{cases}
$$

$$\partial_t \mathbf{w} + \partial_x \mathbf{F}(\mathbf{w}) + \mathbf{B}(\mathbf{w})\partial_x \mathbf{w} = \mathbf{S}(\mathbf{w})\partial_x H + \mathbf{G}(\mathbf{w}),$$

$$r_{j,\alpha} := \phi_{j,\alpha} h \text{ for } j = 0, 1, \ldots, N$$

$$q_\alpha := \rho_\alpha(\Phi) h u_\alpha,$$

$$\mathbf{w} := (h, \mathbf{q}, \mathbf{r})^{\mathrm{T}} \in \mathbb{R}^{((N+1)M+1)}$$

$$m_\alpha = \rho_\alpha(\Phi)h.$$

$$\widetilde{\mathbf{w}} := (h, \mathbf{m}, \mathbf{q})^{\mathrm{T}} \in \mathbb{R}^{2M+1}$$

$$m_\alpha = \rho_\alpha(\Phi) h.$$

$$\begin{cases} \partial_t h + \partial_x \left(h \sum_{\beta=1}^{M} l_\beta u_\beta \right) = 0, \\ \partial_t m_\alpha + \partial_x q_\alpha = \dfrac{1}{l_\alpha} \sum_{j=0}^{N} \rho_j \Delta_-^\alpha H_{j,\alpha+1/2}(\Phi), \\ \partial_t q_\alpha + \partial_x \left(\dfrac{q_\alpha^2}{m_\alpha} \right) + \partial_x \left(gh \left[\dfrac{1}{2} l_\alpha\, m_\alpha + \sum_{\beta=\alpha+1}^{M} l_\beta\, m_\beta \right] \right) \\ \quad = \sum_{j=0}^{N} \dfrac{\rho_j}{l_\alpha} \Delta_-^\alpha (u_{\alpha+1/2} H_{j,\alpha+1/2}(\Phi)) + \left(p_{\mathrm{S}} + gh \sum_{\beta=\alpha+1}^{M} l_\beta \rho_\beta(\Phi) \right) \partial_x h \\ \quad\quad - gh\rho_\alpha \left(\partial_x z_{\mathrm{B}} + \sum_{\beta=1}^{\alpha-1} l_\beta \partial_x h \right). \end{cases}$$

$$\widetilde{\mathbf{w}} := (h, \mathbf{m}, \mathbf{q})^{\mathrm{T}} \in \mathbb{R}^{2M+1}$$

$$m_\alpha = \rho_\alpha(\Phi)h.$$

$$\partial_t \widetilde{\mathbf{w}} \ + \ \partial_x \widetilde{\mathbf{F}}(\widetilde{\mathbf{w}}) + \widetilde{\mathbf{B}}(\widetilde{\mathbf{w}})\partial_x \widetilde{\mathbf{w}} \ = \ \widetilde{\mathbf{S}}(\widetilde{\mathbf{w}})\partial_x \widetilde{H} + \widetilde{\mathbf{G}}(\widetilde{\mathbf{w}}),$$

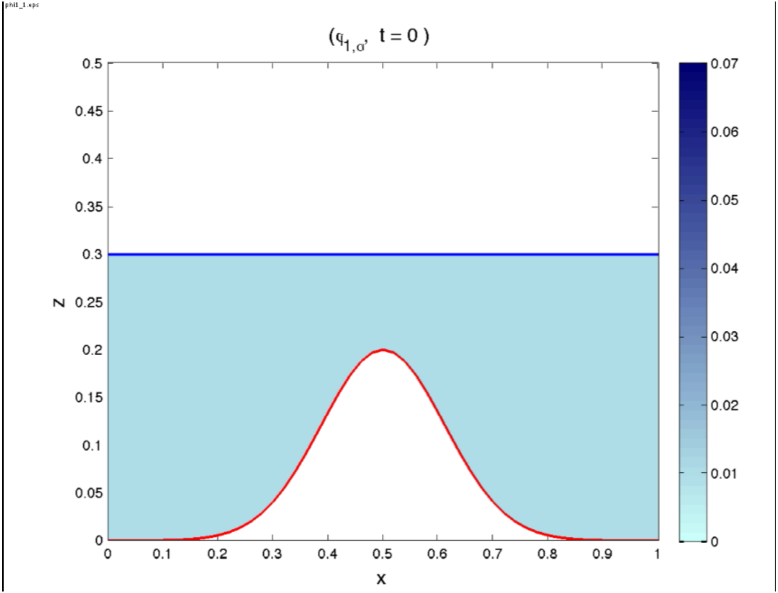
$$\widetilde{\mathbf{w}} := (h, \mathbf{m}, \mathbf{q})^{\mathrm{T}} \in \mathbb{R}^{2M+1}$$

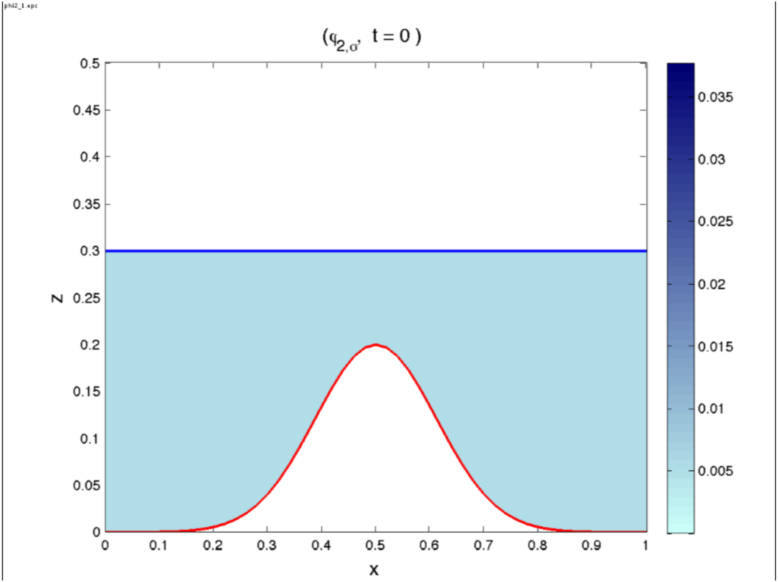
Outline

Numerical scheme

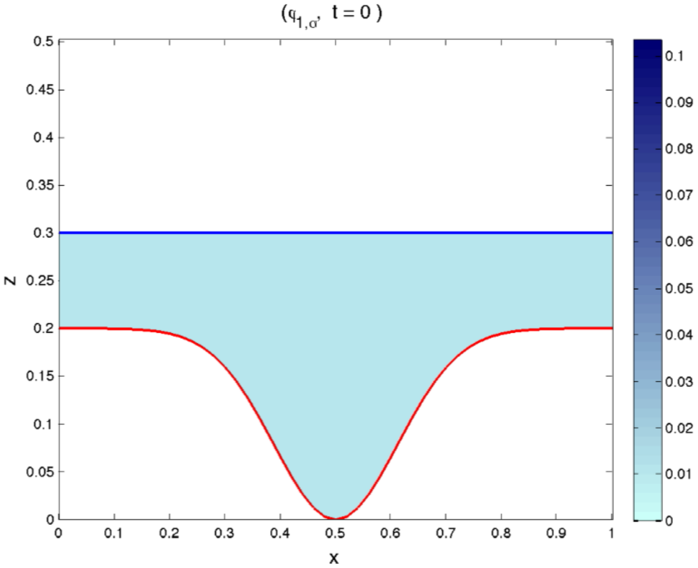
Numerical scheme

Move horizontally then vertically !


phi1_1.eps
$(q_{1,\alpha},\ t = 0)$
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
z
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0

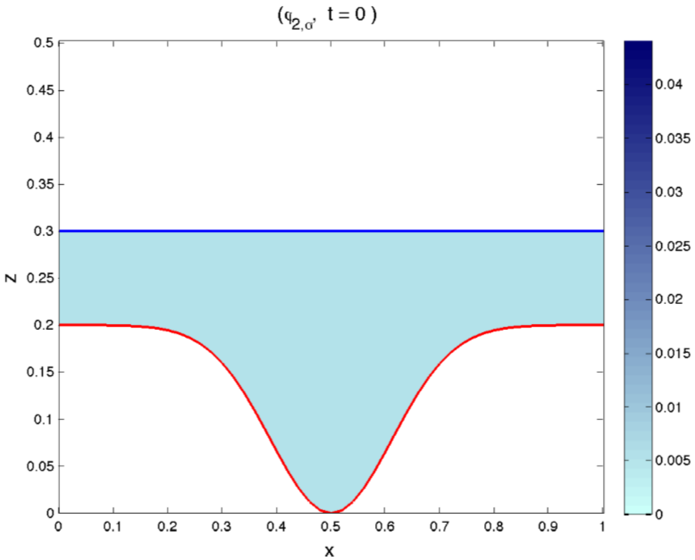

phi2_1.eps
$(q_{2,\sigma},\ t=0)$
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
z
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
0.035
0.03
0.025
0.02
0.015
0.01
0.005

phi1_1.eps



(q_{1,σ}, t = 0)
z
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.05
0.15
0.25
0.35
0.45
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09

phi2_1.eps



$(q_{2,\sigma},\ t=0)$
z
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.05
0.15
0.25
0.35
0.45
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

E.D. Fernández-Nieto, E.H. Koné, T. Morales de Luna, R. Bürger (2013).
A multilayer shallow water system for polydisperse sedimentation
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Thank you for your attention

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