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# A Two-Phase Solid/Fluid Model for Dense Granular Flows including Dilatancy Effects

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# Modeling of debris flows (grain/fluid)

Solid volume fraction: $0.4 < \varphi < 0.8$

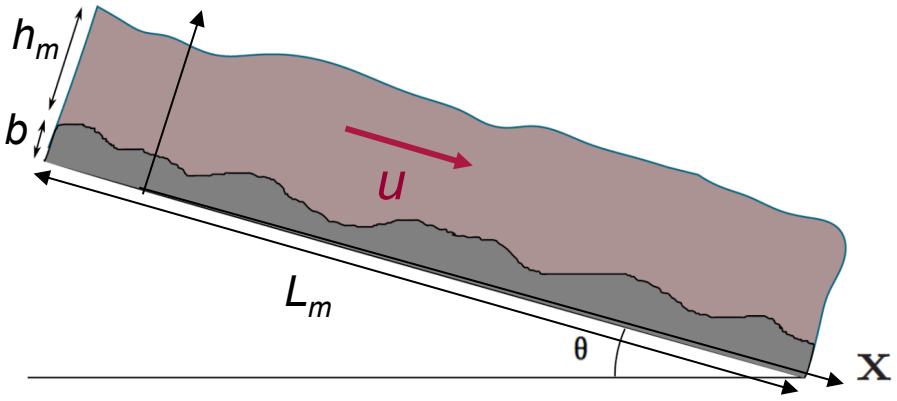
**At the field scale**

- Thin layer approximation $hm/Lm = \varepsilon \ll 1$

$$h_m = \mathcal{O}(\epsilon), \nabla_{\mathbf{x}} b = \mathcal{O}(\epsilon), u^{\mathbf{x}} = \mathcal{O}(1),$$
$$u^z = O(\epsilon), \ldots$$

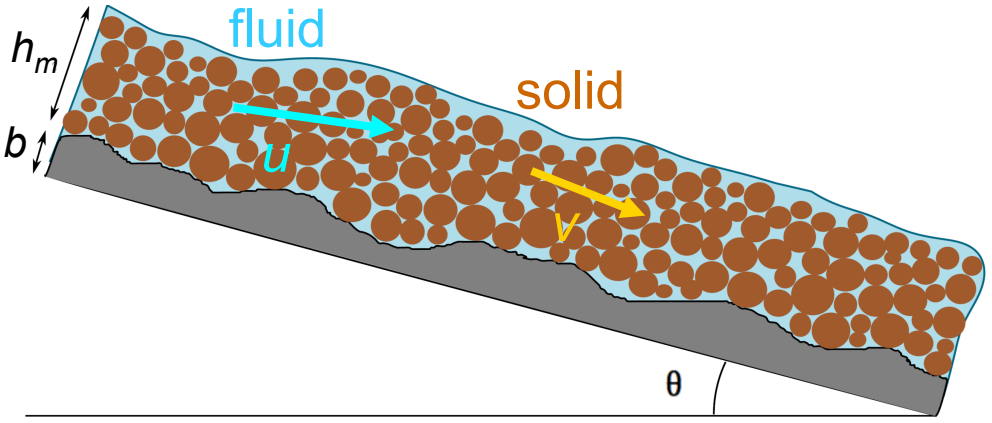
- Depth-averaged model

Mixture model




*Iverson, Denlinger; Denlinger, Iverson 2001, Iverson, George; George, Iverson 2014*

Two-phase model




*Pitman and Le 2005, Pelanti et al. 2008, 2011*

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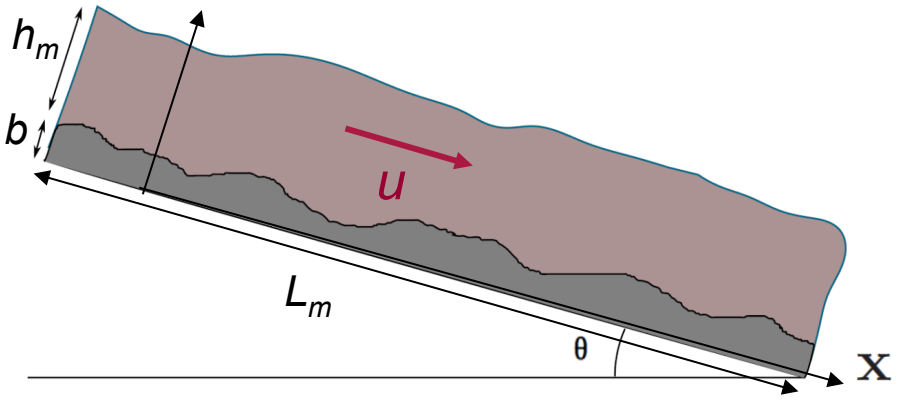


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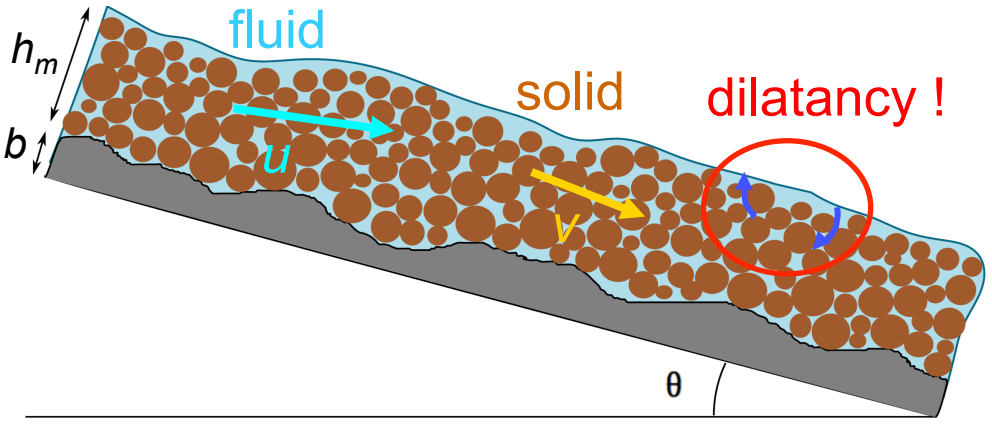


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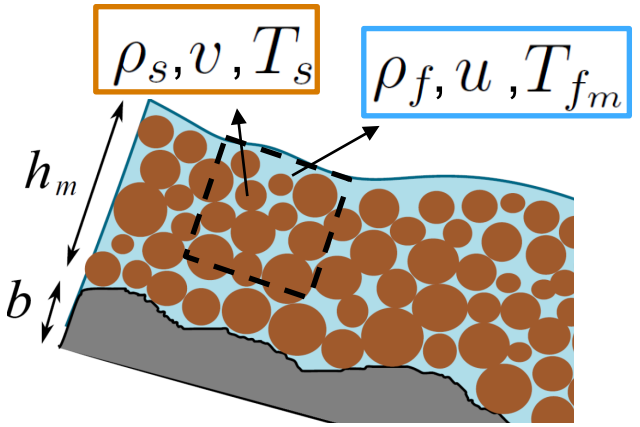
# Jackson's model

*Jackson, 2000*

$\varphi$ : solid volume fraction, $1-\varphi$ : fluid volume fraction




- Mass conservation :
  - $\partial_t(\rho_s\varphi) + \nabla\cdot(\rho_s\varphi v) = 0$
  - $\partial_t(\rho_f(1-\varphi)) + \nabla\cdot(\rho_f(1-\varphi)u) = 0$
- Momentum conservation :
  - $\rho_s\varphi(\partial_t v + (v\cdot\nabla)v) = -\nabla\cdot T_s - \varphi\nabla p_{f_m} + f + \rho_s\varphi\mathbf{g}$
  - $\rho_f(1-\varphi)(\partial_t u + (u\cdot\nabla)u) = -\nabla\cdot T_{f_m} + \varphi\nabla p_{f_m} - f + \rho_f(1-\varphi)\mathbf{g}$

$$T_s = p_s\,\mathrm{Id} + \widetilde{T}_s \qquad T_{f_m} = p_{f_m}\,\mathrm{Id} + \widetilde{T}_{f_m}$$

Friction between the solid and fluid phases : $f = \beta(u-v)$

5 unknowns : $\varphi, v, u, p_s, p_{f_m}$, 4 equations

**A constitutive equation is required to close the system**

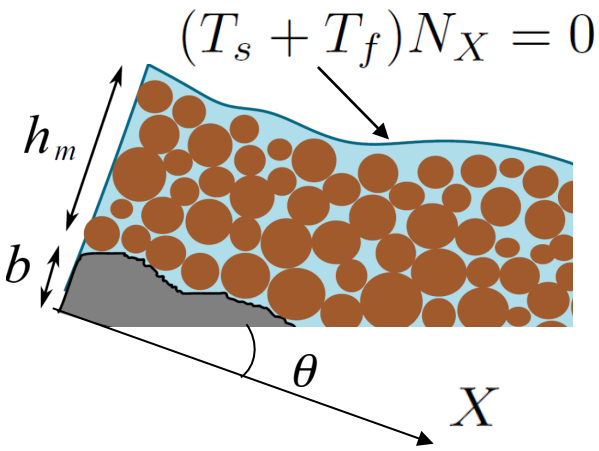
# Closure equation and energy dissipation

- Former thin layer depth-averaged models *(Pitman and Le 2005, Pelanti et al. 2008)*

Additional **free surface boundary condition** instead of a closure equation:

$$p_s = p_{f_m} = 0$$

Hydrostatic pressure + depth-averaging
➡ boundary condition ≈ closure inside the domain




➡ Dissipation of energy is not ensured !

- **FIRST STEP** closure equation : Incompressibility of the solid phase

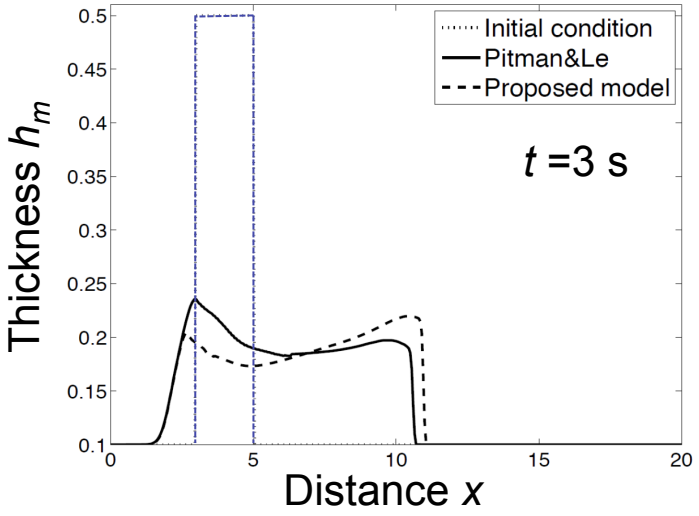
$$\nabla \cdot v = 0$$

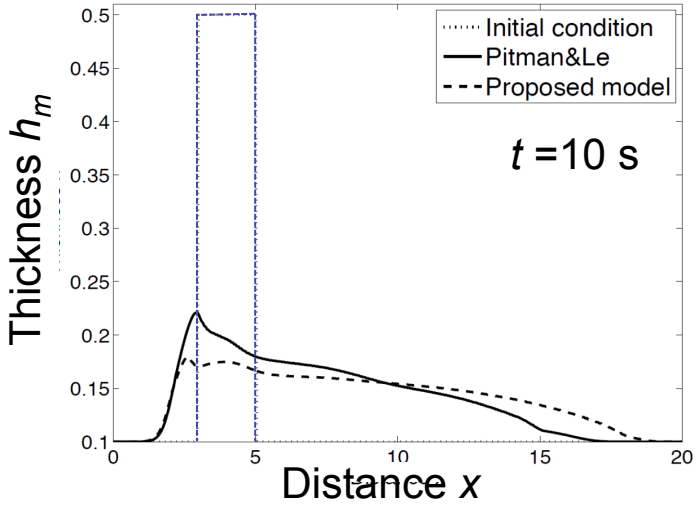
**Dissipative energy equation**

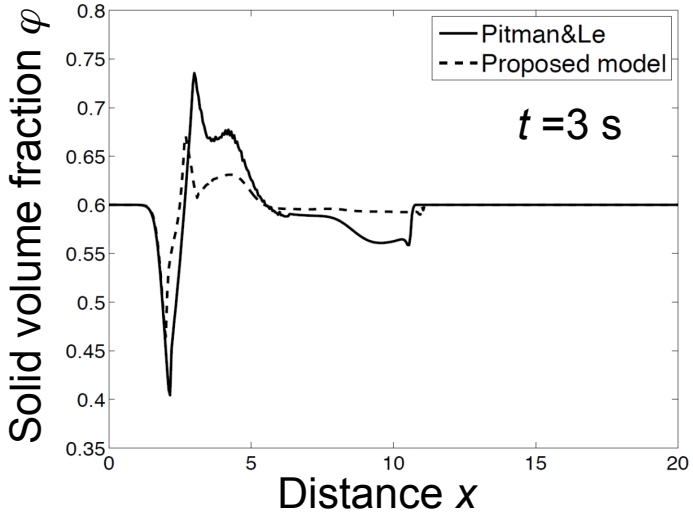
*Bouchut, Fernandez-Nieto, Mangeney, Narbona-Reina, 2015*

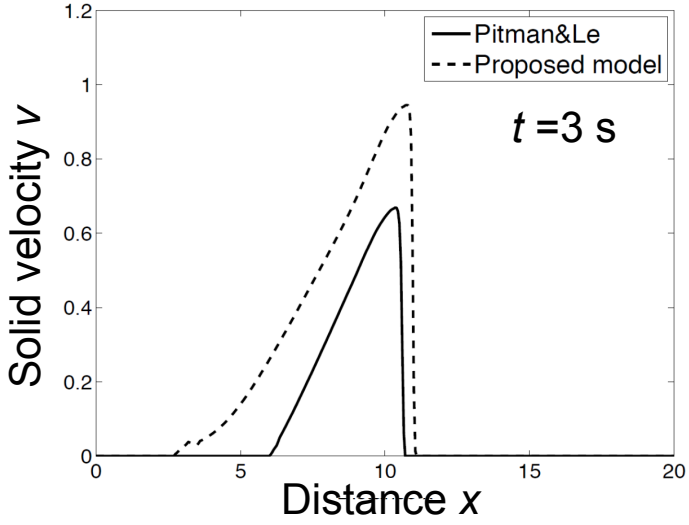
# Difference with Pitman and Le's model

Initial volume fraction : $\varphi_0 = 0.6$, slope angle $\theta = 10^o$




*Bouchut, Fernandez-Nieto, Mangeney, Narbona-Reina, 2015*

# Qualitative explanation of dilatancy effects

Critical state : $\varphi_c^{eq}$ *when deformed*

$\varphi < \varphi_c^{eq}$ (loose)

Granular medium contracts

⬇

Fluid is expelled ( $\mathcal{V}_f$ )

⬇

Fluid pore pressure increases

⬇

Liquefaction

$\varphi > \varphi_c^{eq}$ (dense)

Granular medium dilates

⬇

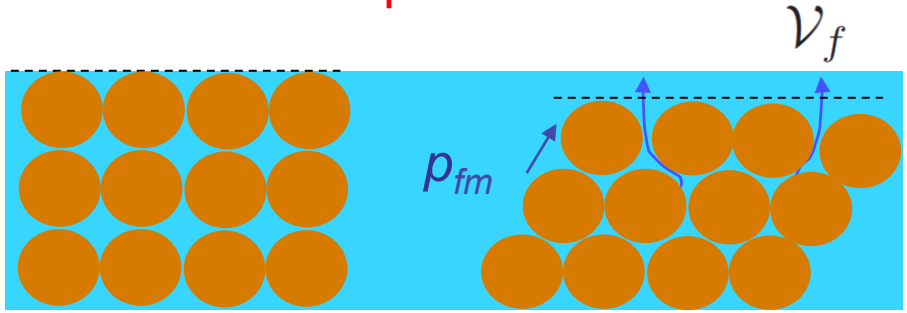
Fluid is sucked (-$\mathcal{V}_f$)

⬇

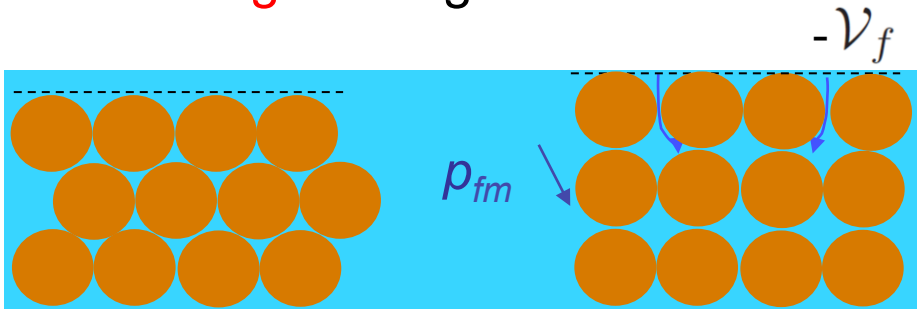
Fluid pore pressure decreases

⬇

Stiffening of the granular matrix




Excess pore pressure $p^e_{fm} \geq 0$




$p^e_{fm} \leq 0$

Coulomb friction: $T^{\mathbf{x}z}_{s\,|b} = \tan\overline{\delta_{\text{eff}}}\left(\bar{\varphi}(\rho_s - \rho_f)g\cos\theta h_m - (p^e_{f_m})_{|b}\right)$

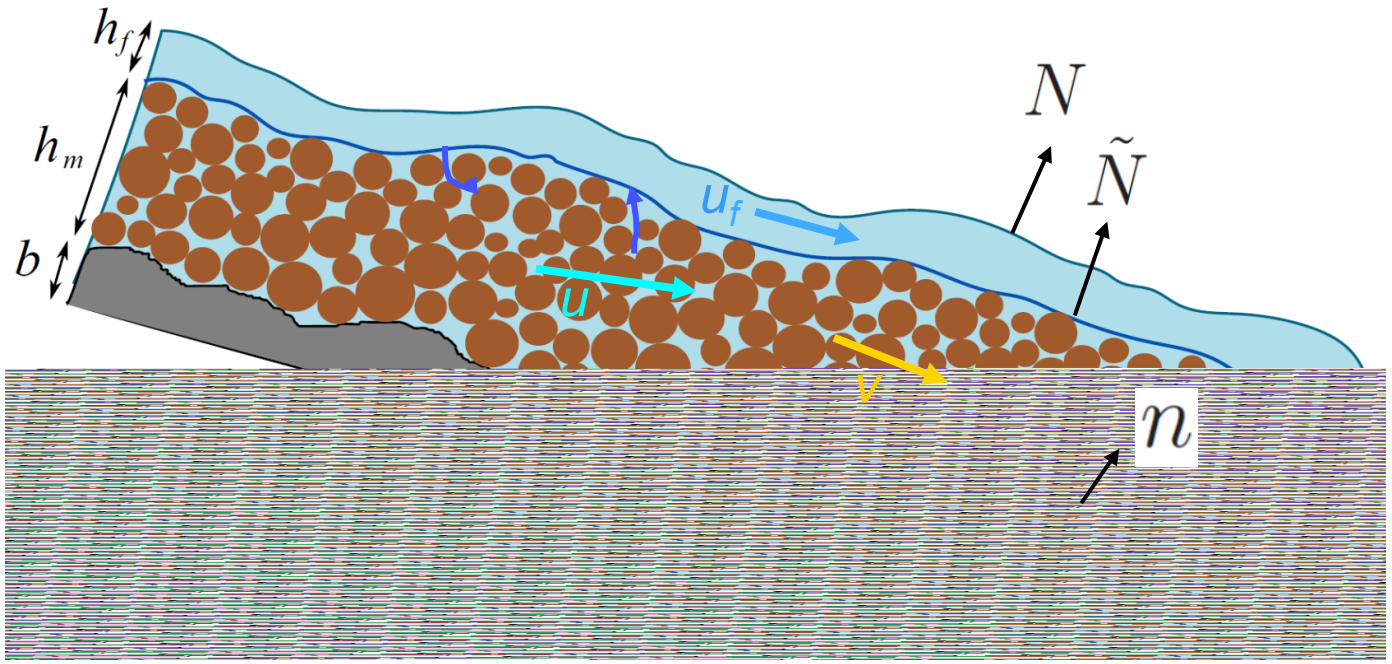
*e. g. Schofield and Wroth, 1968, Wood, 1990, Pailha and Pouliquen, 2009*

# Differential motion of the fluid and solid phases

A crucial element :

Make it possible for the fluid to enter/escape the granular phase

Solid free surface ≠ fluid free surface !

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