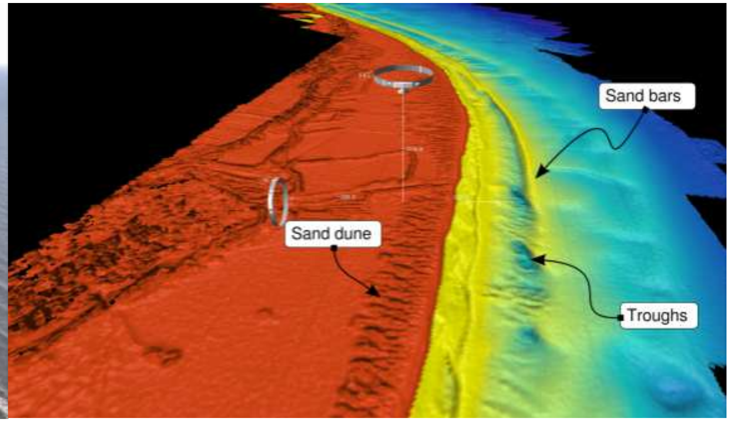


Uncertainty quantification in littoral erosion

Ingredients

+ Morphodynamics by minimization principle

+ Design of defense structures and coastal engineering

+ Uncertainties on bed characteristics

+ Uncertainties on the state

+ Extreme scenarios

Rappel fonctionnalités de la plate-forme :

-Saint-Venant+Bathy dynamique / minimisation d'énergie

-Optimisation de formes, adjoint auto par tapenade

-Scénarii extrêmes (Quantiles)

-UQ

Evolution par rapport à l'année dernière :

-introduction de la vitesse orbitale équivalente

- lien avec l'infragravité

- amélioration de la partie incertitude

Fluid/Bottom model

Dependency chain: $\psi \rightarrow \{\mathbf{U}(\psi,\tau), \tau \in [0,T]\} \rightarrow J(\psi,T)$

Flow state equation: $\mathbf{U}_t + F(\mathbf{U},\psi) = 0, \quad \mathbf{U}(0) = \mathbf{U}_0(\psi)$

Cost fct: $J(\psi,T) = \int_{(0,T)} j(\psi, \mathbf{U}(\psi,t))$

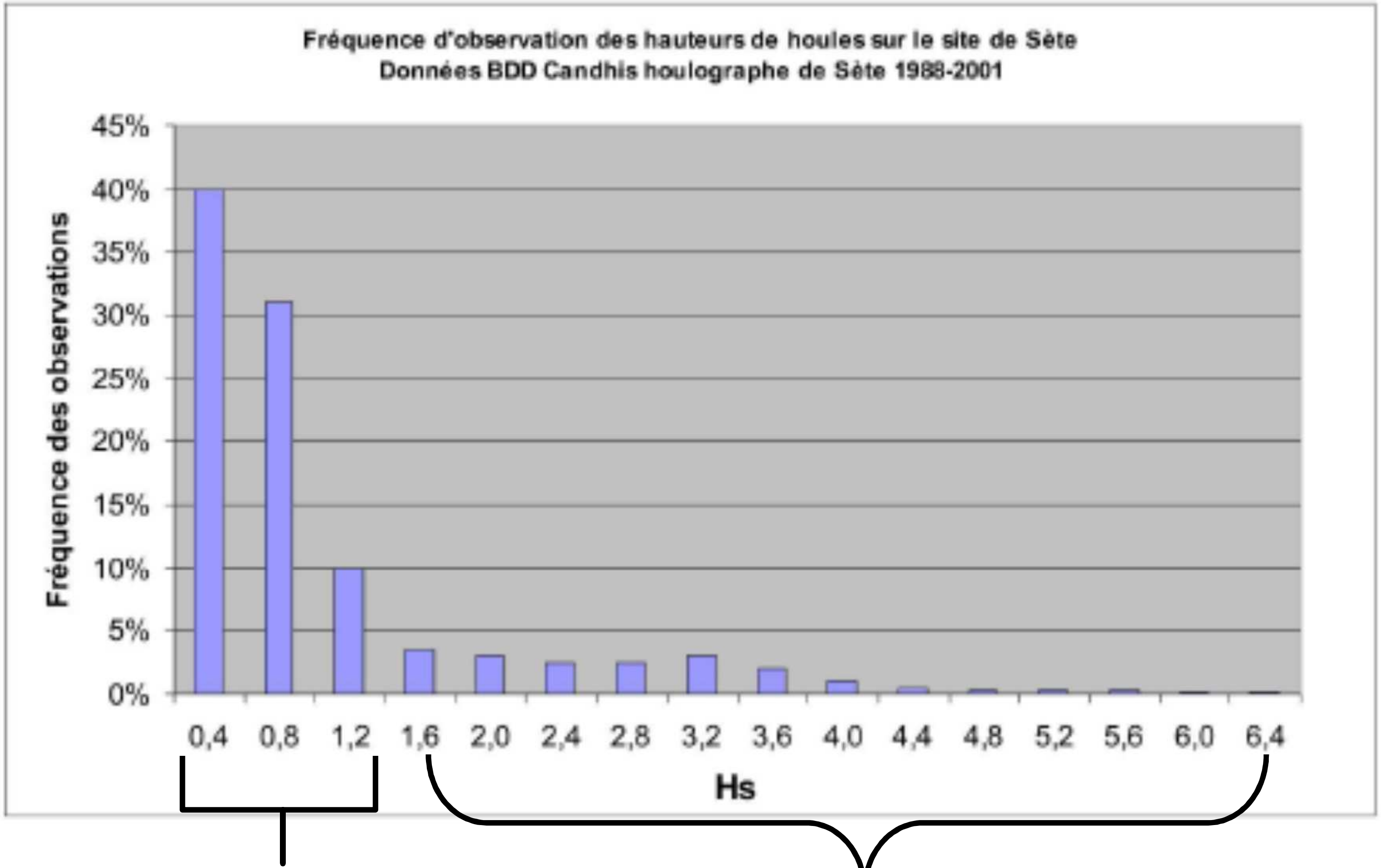
Bed motion: $\partial_t \psi = -\rho(t,x)\nabla_\psi J, \quad \psi(t=0,x) = \psi_0(x) =$ given,

Bed time and space variability through its response to flow perturbations.
Aleatory uncertainties also present in initial and boundary conditions.
Epistemic uncertainties due to model & numerics.

→ Same platform used to design beach protection devices (geotube, sand dune, groyne, etc).

→ Long term experience with automatic differentiation in reverse mode

Erosion and waves



Constructive waves

Destructive waves

Ansatz

The bed adapts in order to reduce water kinetic energy (do know how !)
with 'minimal' sand transport

We do not know details of microscopic mechanisms.
We are interested by macroscopic features.

Example of functional

for beach morphodynamics simulations

T: Time interval of influence
Ω *:* observation domain

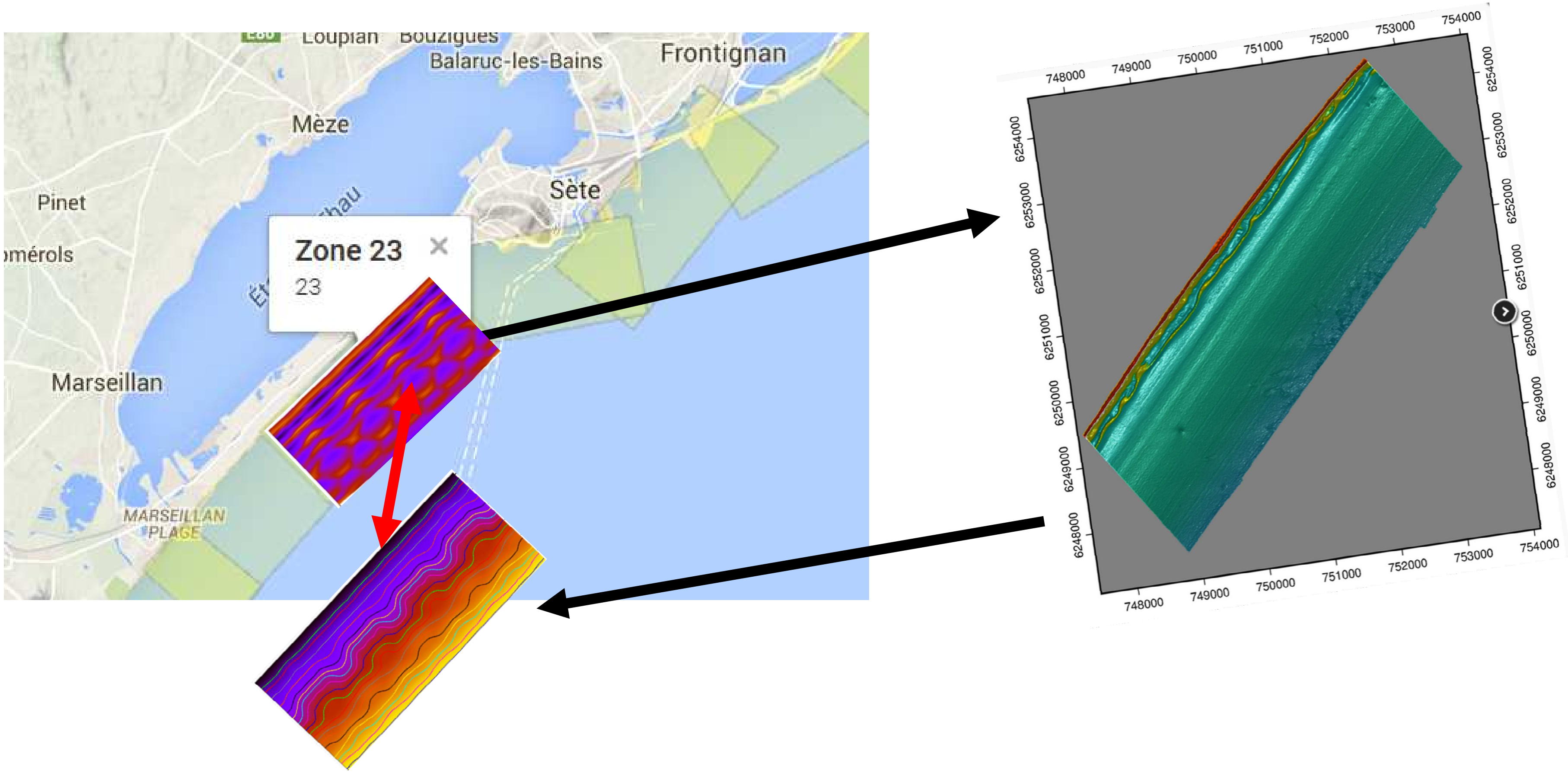
$$J(U(\psi)) = \int_{t-T}^{t} \int_{\Omega} \left(\frac{1}{2}\rho_w g \eta^2 + \rho_s g(\psi(\tau) - \psi(t-T))^2\right) d\tau d\Omega$$

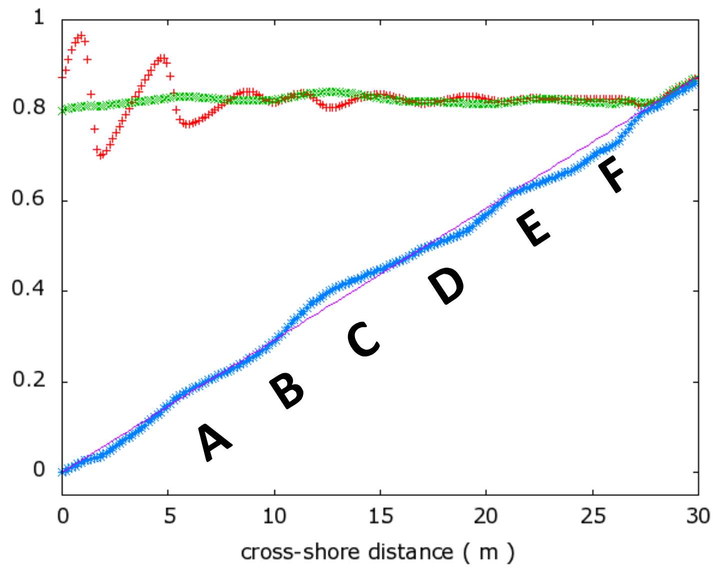
$$\eta(\tau, x, \psi) = h(\tau, x, \psi) - \frac{1}{T}\int_{t-T}^{t} h(\tau, x, \psi) d\tau$$

Phd Afaf Bouharguane

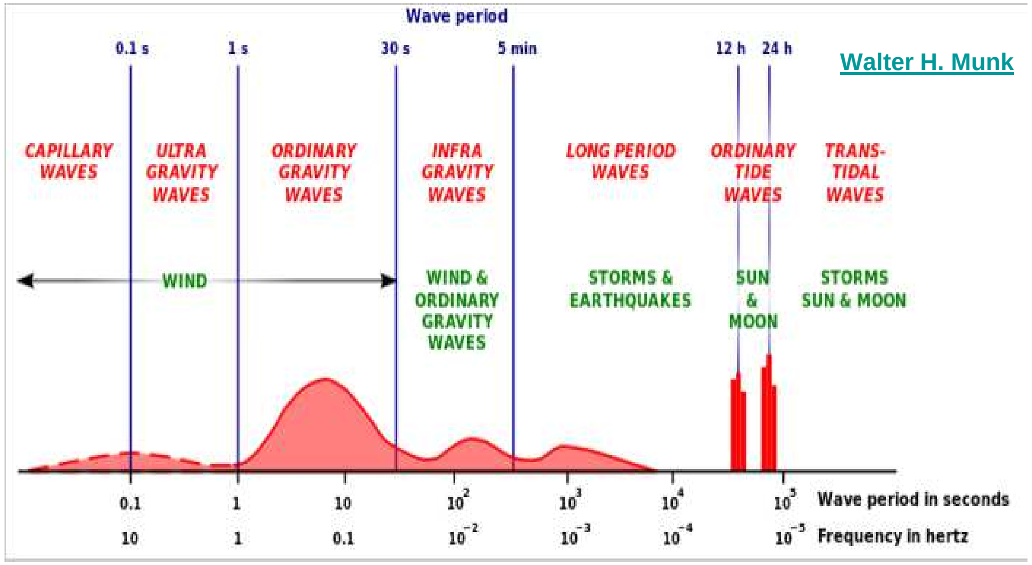
Coupling water motion & bottom

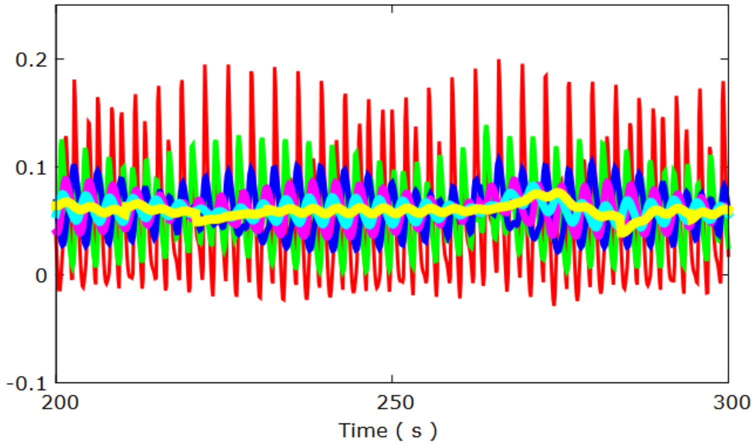
http://www.soltc.org/database/lidar

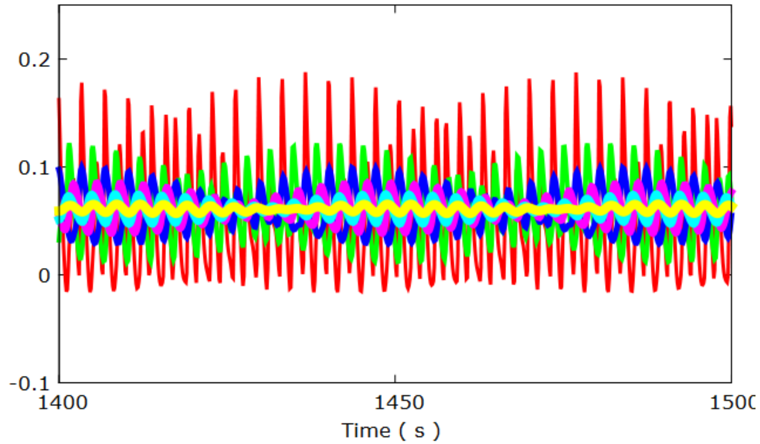




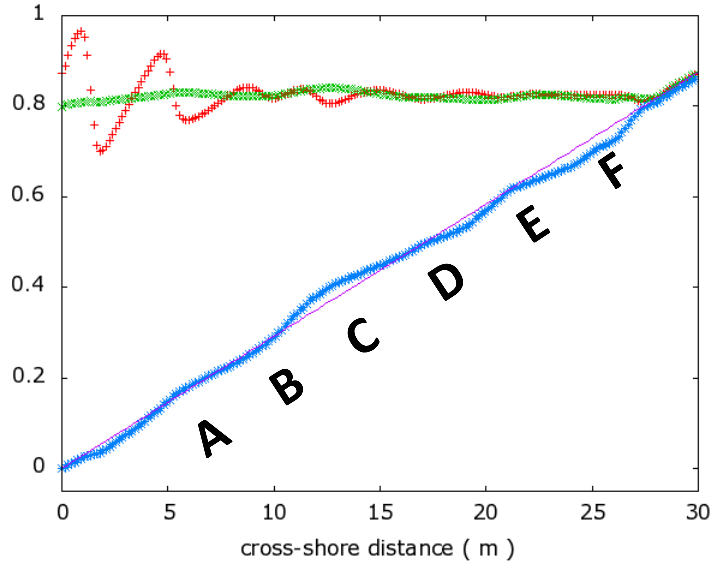
Impact on local elevations in time





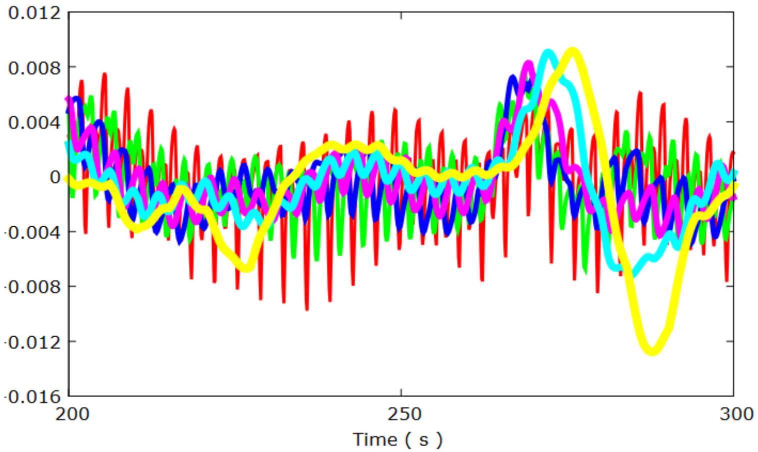


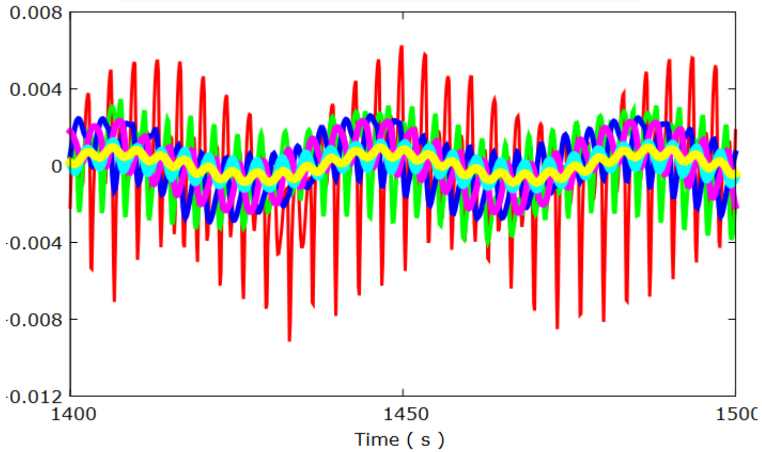
Impact of bathy changes on water elevation



Impact on infragravity waves (0.004-0.04 Hz)

Surf can be seen breaking as it crosses the sand bar offshore. Sandbars aid in generating infragravity waves and in turn are shaped by them.





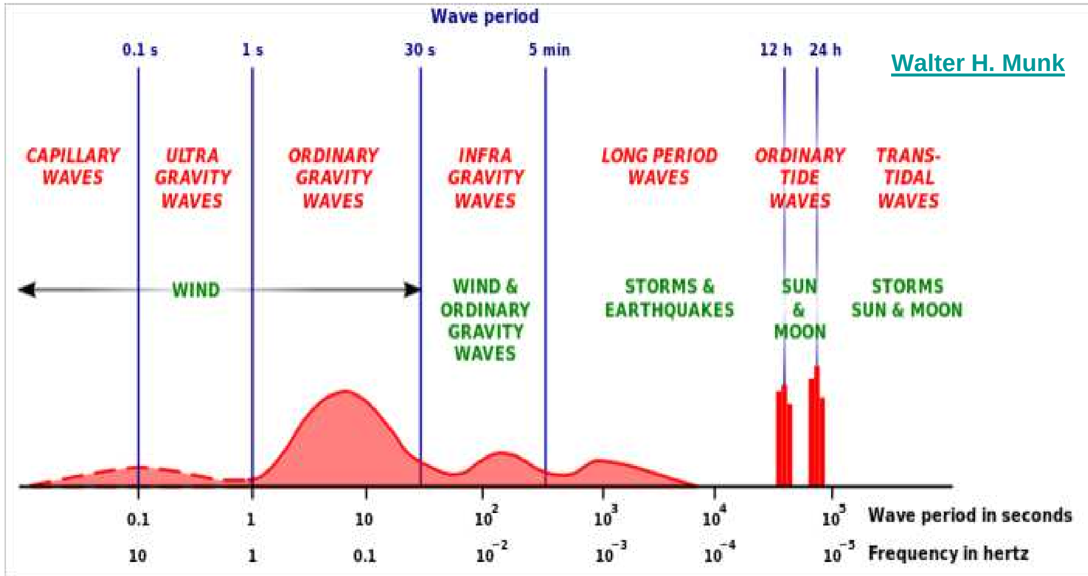
Use infragravity waves in the functional

Swell memory

Infragravity waves need long time integration

Difficulty with time dependent adjoint

Need to express/control infragravity through gravity information



Forward propagation

Impact of bed characteristics uncertainty

Forward propagation

Randomness well located in the simulation chain.

$$\psi \longrightarrow u(\psi) \longrightarrow j(u(\psi))$$

PDF of ψ known and in particular its VaR

Want to avoid any sampling of the parameter space and propagation of the uncertainty in the simulation chain.

Introduce

VaR-based directional extreme scenarios.

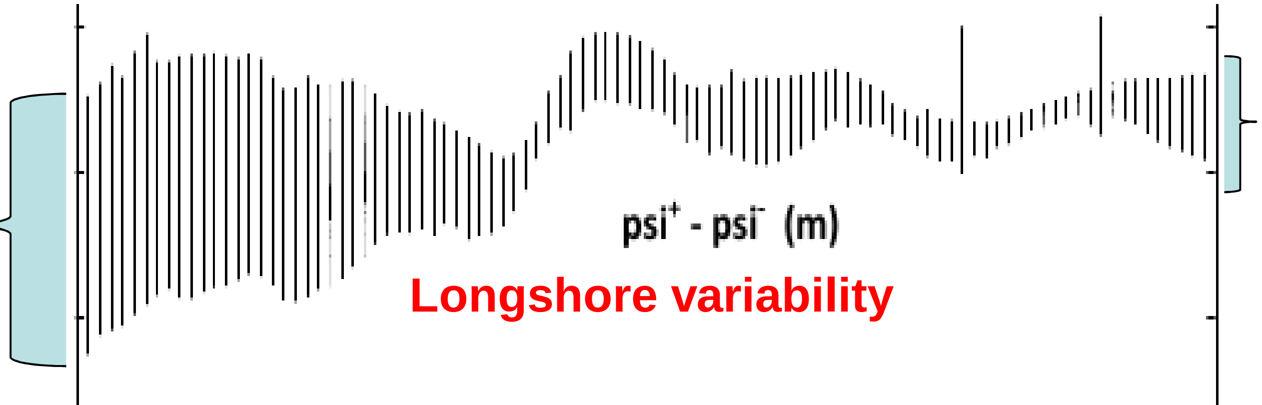
Bed variability due to bed mobility variability

Assumption: sand mobility variability increases toward the beach: from 0 to 50%

This also accounts for imperfect modelling (epistemic UQ)

Water depth decreases from 5 m to 20 cm

~10cm Variability for 5m water depth: 2%



~5cm Variability for 20cm water depth: 25%

Cross-shore distance: 150m

Reverse propagation

Impact of state/observation uncertainty

- Directional Extreme Scenarios (DES)

Covariance matrix by:
- Cumulative adjoint based construction

- Ensemble analysis (EnKF / UKF)
→ Ensemble DES

-2015, Backward uncertainty propagation in shape optimization, IJNMF.
-2015, Ensemble Kalman Filters and geometric characterization of sensitivity spaces for UQ, CMAME.
-2016, Uncertainty quantification in littoral erosion, C&F

Adjoint-based variance estimation
linear relationship between control and state

$u = J\,x$ → $Cov(x) = (J^t\,Cov(u)^{-1}\,J)^{-1}$

Cumulative construction

$$\mathbb{E}(u) = \mu,\ u = \mu + \delta u,\ \delta u = J\ \delta x,\ J = \nabla_x u$$

$$Cov(u) = \mathbb{E}(uu^t) - \mu\mu^t$$
$$= 2\mathbb{E}(u\ \delta u^t) + \mathbb{E}(\delta u\ \delta u^t) = \mathbb{E}(\delta u\ \delta u^t) \quad \text{(H1)}$$

$$Cov(u) = J\ \mathbb{E}(\delta x\ \delta x^t)\ J^t$$
$$Cov(\delta x) = J^{-1}\ Cov(u)\ J^{-t} = (J^t\ Cov(u)^{-1}\ J)^{-1}$$

$$Cov(\delta x) = \mathbb{E}(\delta x\ \delta x^t) - \mathbb{E}(\delta x)\ \mathbb{E}(\delta x)^t$$
$$\mathbb{E}(\delta x) = 0 \quad \text{(H2)}$$

$$Cov(x + \delta x) = Cov(x) + Cov(\delta x) + 2Cov(x, \delta x)$$
$$\sim Cov(\delta x) \quad \text{(H3)}$$

where $\nabla_x j$ by adjoint and $\mathcal{J} = (\frac{\partial j}{\partial u})^{-t}\ \ (\nabla_x j - \frac{\partial j}{\partial x})^t$

Directional Extreme Scenarios to estimate the robustness of the design

Two-step construction

1/ $x^* = \arg\min_x (j(u(x)))$

2/ $(x^*)^{+/-}$ with $j \leftarrow j(u(x)) + \left\| u(x) - (u^*)^{+/-} \right\|$

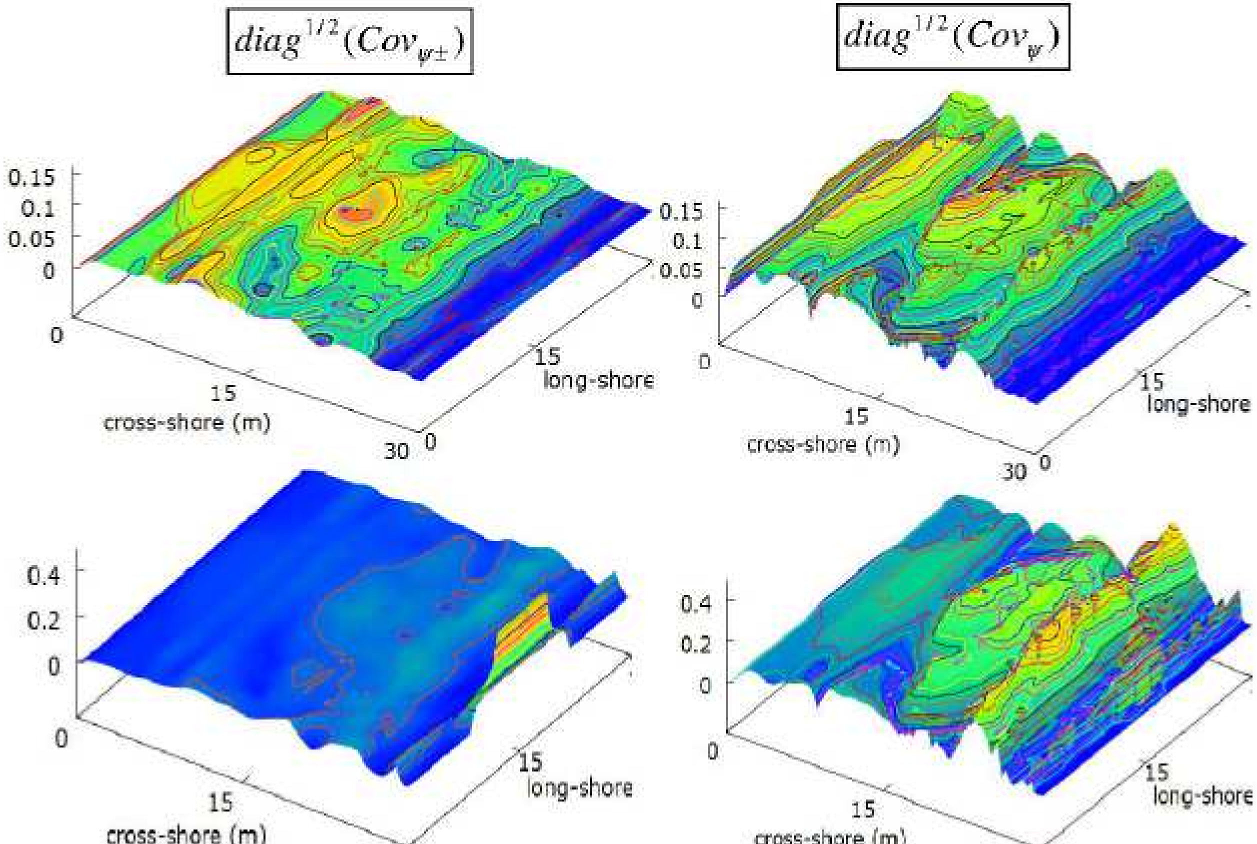
$(u^*)^{+/-}$: VaR based extreme scenarios

Cov_{u^*} known & modelling

aleatoric and epistemic uncertainties

x^* & $(x^*)^{+/-}$ give an indication on the robustness of the design

Bed covariance
Extreme scenarios vs. Adjoint-based



Relative variability

State uncertainty induces
large variability in swash zone,
up to 40% of the predicted bathymetry.

Remarks

- Morphodynamics by minimization principle

-Quantile-based directional extreme scenarios

-Different bed covariance estimations
accounting for epistemic & aleatory state variability

- Open sea not basin

-Applications: Aeronautics (Dassault Aviation, Airbus),
seismic (TOTAL, GM, Labex Numev),
Biomed network circulatory systems (CHU MTP),
Littoral (GDR Egrin, GM), Thin films (GDR Fast)