A numerical scheme for a two dimensional non-hydrostatic shallow water system.

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Abstract

We present a numerical method for a two dimensional dispersive shallow water system with topography. The objective is to capture dispersive effects due to a non-hydrostatic contribution in a model derived from a depth averaged Euler incompressible system with free surface (see [3] for the derivation). The method is based on an operator splitting procedure in the spirit of the prediction-correction method initially introduced by Chorin-Temam (see [4]). Then, we propose a variational formulation of the correction step and we apply a finite element method with compatible spaces to the two dimensional problem on unstructured grids. This method is an extension of the one dimensional method which has been developed in [1] and [2] with classical mathematical properties. A numerical treatment when H tends to zero is also provided to ensure the ability of the method to treat wet/dry transitions, which is crucial in geophysical flow. Comparisons with analytical solutions and classical test cases are performed to evaluate the efficiency of our method.

References

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