

# Dynamic behaviour of the static/flowing interface in viscoplastic granular flows: analytic, numerical and experimental studies

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**Keywords:** Viscoplastic granular flows, static/flowing interface, granular collapse

Observed avalanche flows of dense granular material have the property to present two possible behaviours: static (solid) or flowing (fluid). In such situation, an important challenge is to describe by differential equations the evolution of the physical interface between the two phases. Defining an equation for this interface in thin-layer models has been done based on phenomenological models or on strong assumptions such as a specified velocity profile or reducing the flow to a sliding block for example in [1, 2]. A review on static/flowing models can be found in [3].

In this talk we would like to describe the evolution of the static/flowing interface via a set of equations derived analytically via an asymptotic expansion starting from a viscoplastic model with Drucker-Prager yield stress, analogous to the  $\mu(I)$  rheology of [4]. A set of equations has been derived in [5], and assumes a thin-layer regime. The model takes the form of a formally overdetermined initial-boundary problem in the variable normal to the topography (that does not disappear even with the thin-layer assumption), set in the flowing region only. The extra boundary condition at the static/flowing interface involves a third order derivative of the velocity in the normal direction, and contains the information on how to evolve the static/flowing interface, but in a non explicit manner. It comes out from the continuity of the velocity and shear stress across the interface. The model handles arbitrary velocity profiles, and is therefore more general than classical depth-averaged models. Explicit solutions can be built [6] and show different possible behaviours such as progressive stopping or sudden start of a part of the material. We have performed several studies [7, 8] to numerically resolve our set of equations and compare the results to data from laboratory experiments of granular collapse from [9] and to two-dimensional simulations as those in [10].

## References

- [1] I.S. Aranson, L.S. Tsimring, *Continuum theory of partially fluidized granular flows*, Phys. Rev. E **65**, 061303 (2002).
- [2] D.V. Khakhar, A.V. Orpe, P. Andresen, J.M. Ottino, *Surface flow of granular materials: model and experiments in heap formation*, J. Fluid Mech. **441**, 225-264 (2001).
- [3] R.M. Iverson, C. Ouyang, *Entrainment of bed material by Earth-surface mass flows: review and reformulation of depth-integrated theory*, Rev. Geophys. **53**, 27-58 (2015).
- [4] P. Jop, Y. Forterre, O. Pouliquen, *A constitutive law for dense granular flows*, Nature **441**, 727-730 (2006).
- [5] F. Bouchut, I.R. Ionescu, A. Mangeney, *An analytic approach for the evolution of the static/flowing interface in viscoplastic granular flows*, Comm. Math. Sci **14** (2016).
- [6] C. Lusso, F. Bouchut, A. Ern, A. Mangeney, *Explicit solutions to a free boundary model for the static/flowing transition in granular flows*, submitted, 2015.
- [7] C. Lusso, F. Bouchut, A. Ern, A. Mangeney, *A simplified model for static/flowing dynamics in thin-layer flows of granular materials with yield*, submitted, 2015.
- [8] C. Lusso, A. Ern, F. Bouchut, A. Mangeney, M. Farin, O. Roche, *Two-dimensional simulation by regularization of free surface viscoplastic flows with Drucker-Prager yield stress and application to granular collapse*, submitted, 2015.
- [9] M. Farin, A. Mangeney, O. Roche, *Fundamental changes of granular flow dynamics, deposition, and erosion processes at high slope angles: Insights from laboratory experiments*, J. Geophys. Res. Earth Surf. **119**, 504-532 (2014).
- [10] I.R. Ionescu, A. Mangeney, F. Bouchut, O. Roche, *Viscoplastic modeling of granular column collapse with pressure-dependent rheology*, J. Non-Newtonian Fluid Mech. **219**, 1-18 (2015).